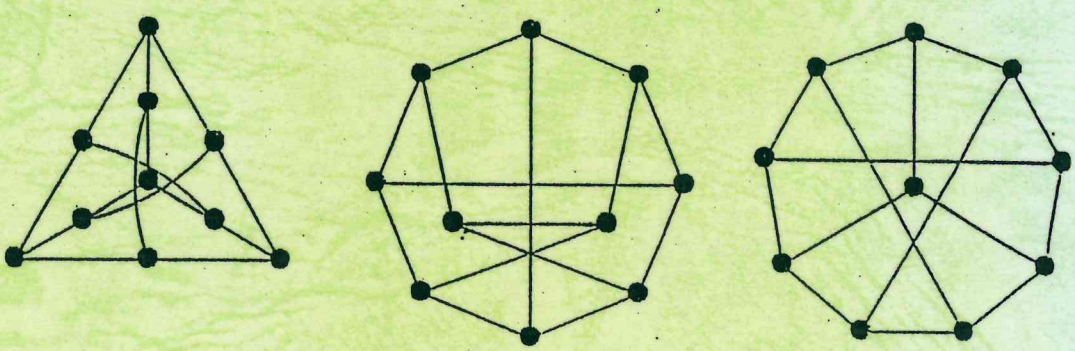
組合數學新苗研討會

論文摘要

Workshop on Combinatorics for Newly Graduated Students

June 16 ,2000



主辦單位：逢甲大學應用數學系

協辦單位：行政院國科會數學中心

時　　間：2000 年 6 月 16 日

地　　點：台中市逢甲大學理學院

A mathematician is a machine for turning coffee into theorems.

---- *Paul Erdős*

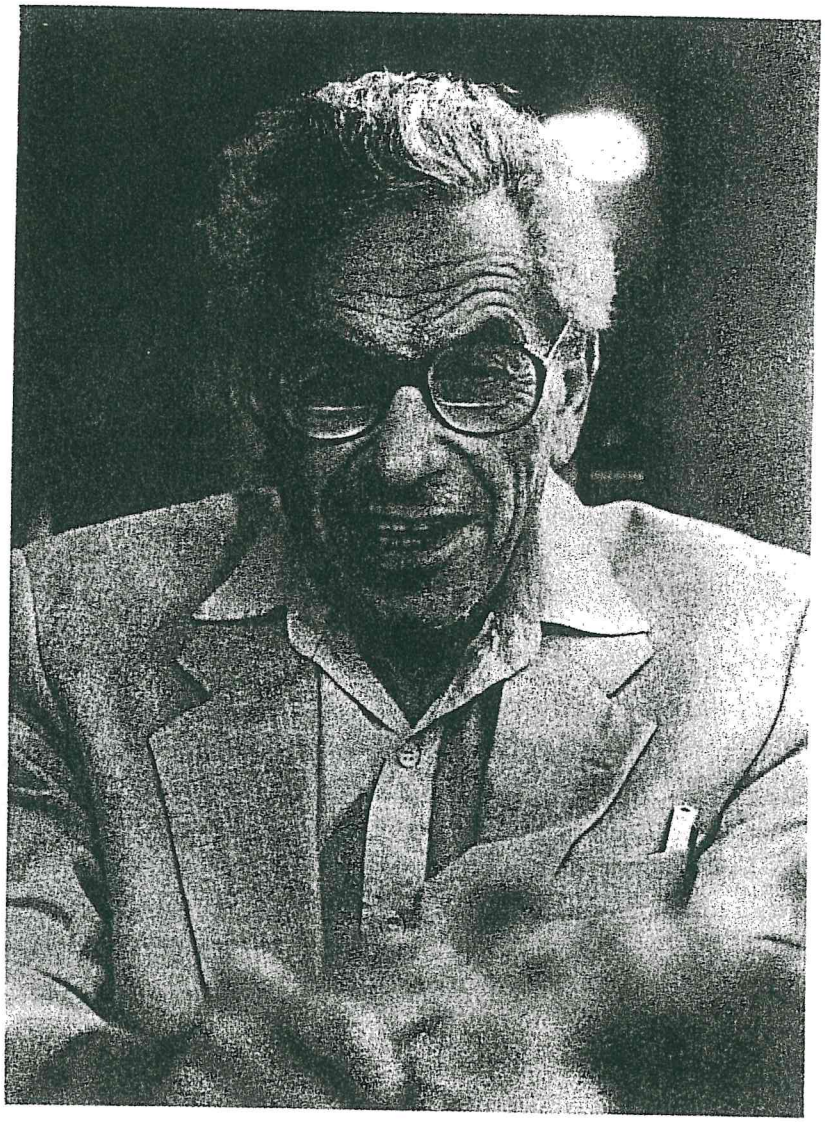


Photo by George Paul Csicsery from his documentary film *N is a Number: A Portrait of Paul Erdös* (1993).

2000 組合數學新苗研討會時程表

6 月 16 日

上午

10:00-10:20　報到

10:20-10:30　開幕

10:30-11:45　Session I

陳永栓(逢甲大學)

A study on graph labellings with two levels of constraint

黃潤吉(逢甲大學)

Graph labeling on the product of two cycles

田互汶(交通大學)

Broadcasting problem in communication networks

11:45-1:30　午餐

下午

1:30-1:40　頒獎 (1999 新苗研討會傑出論文獎)

1:40-2:30　Session II

廖敏如(交通大學)

Two problems in combinatorial optimization

阮夙姿(交通大學)

Group testing problems

2:30-2:50　休息

2:50-4:05　Session III

王治平(交通大學)

Double-loop networks with minimum diameter

張智賢(交通大學)

The reliability of graphs

林瑩貞(台灣大學)

Study of disk graphs

4:05-4:25　休息

4:25-5:40　Session IV

潘志實(中山大學)

Density of the circular chromatic numbers of series-parallel graphs

鄭凱鐘(交通大學)

Total relative displacement of permutation in graph

戴詩珍(交通大學)

On the size of k-forced critical set

目錄

The Reliability of Graphs

Student: Chih-Hsien Chang Advisor: Gerard J. Chang

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National Chaio Tung University

Abstract

Probability graph models have been widely used for the last few decades as models for reliability analysis of computer and communication networks. In this thesis, we study a reliability problem for graphs in which the links are perfectly reliable and the vertices fail independently with known probabilities. More precisely, suppose $G = (V, E)$ is a graph with a given target set $T \subseteq V$ and each vertex $v \in V \backslash T$ has a working probability p_v. The reliability of G is the probability that T is in a component of the subgraph induced by T and the working vertices. Formally, the reliability is

$$\mathrm{Rel}_T(G) = \sum \{ \prod_{i \in S} p_i \prod_{j \notin S \cup T} q_j : S \subseteq V(G) \backslash T \text{ and } T \text{ is in a component of } G[S \cup T]. \}$$

The main results of this thesis is to design efficient algorithms for the problems in block graphs, cactus graphs, and series-parallel graphs.

A Study on Graph Labellings with Two Levels of Constraint

Student : Yong-Shuan Chen Advisor : Roger K. Yeh

Department of Applied Mathematics
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Taichung 40724, Taiwan, R.O.C.

Abstract

For positive integers $a \geq b \geq 1$ and k, a k-$S(a,b)$-*labelling* of a graph G is a function on the vertex set of G, $f : V(G) \to \{0, 1, 2, \cdots, k-1\}$, such that

$$|f(u) - f(v)|_k \geq \begin{cases} a & \text{if } d_G(u,v) = 1, \\ b & \text{if } d_G(u,v) = 2. \end{cases}$$

where $|x|_k = \min\{|x|, k-|x|\}$ is the *circular difference* modulo k. In general, this kind of labelling is called the $S(a,b)$-*labelling* . The $\sigma_{a,b}$-number of G, $\sigma_{a,b}(G)$, is the minimum k of such a k-$S(a,b)$-labelling of G.

We study basic properties of this type of labelling. The $\sigma_{a,b}$-numbers on several classes of graphs are also be investigated.

Total Relative Displacement of Permutation in Graph

Student: Kai-Chung Cheng Advisor: Hung-Lin Fu

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National Chiao Tung University

Abstract

Let α be a permutation of the n vertices of a connected graph G. Define $\delta_\alpha(G)$ to be $\sum_{a,b\in V(G)} |d_G(a,b) - d_G(\alpha(a),\alpha(b))|$, where the sum is over all the $\binom{n}{2}$ unordered pairs of distinct vertices of G. The number $\delta_\alpha(G)$ is called the *total relative displacement* of α in G. So, permutation α is an automorphism of G if and only if $\delta_\alpha(G) = 0$. Let $\pi(G)$ denote the smallest positive value of $\delta_\alpha(G)$ among the n! permutations α of the vertices of G. A permutation α for which $\pi(G) = \delta_\alpha(G)$ has been called a *near-automorphism* of G. Let $\pi^*(G)$ be the maximum value of $\delta_\alpha(G)$ among all permutations of $V(G)$ and the permutation which realizes $\pi^*(G)$ is called a *chaotic mapping* of G.

In this thesis, we study the structure of a connected graph G for $\pi(G) = 2$ and the bound of $\pi^*(G)$ when G is a cycle with n vertices.

Graph Labelling on the Product of Two Cycles

Student : Ruen-Ji Hwang Advisor : Roger K. Yeh

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Abstract

There are considerable articles on the distance two labelling problem. We consider the $L(d,1)$-labelling ($d \geq 2$) on the (Cartesian) product of two cycles.

Two Problems in Combinatorial Optimizaton

Student: Marina M. Liao Advisor: Frank K. Hwang

Department of Applied Mathematics
National Chaio Tung University

Abstract

There are two main parts in this thesis, one is the counterfeit coin's problem, the other is the optimal partition problem.

The first problem is to find the counterfeit coin using a scale through a nonadaptive algorithm. We give the optimal solution of the maximum number of tested coins in t given tests for some t and k. We also provide the construction of testing matrices.

Supermodularity is one of the important properties in optimal partion. In the second part of the thesis, we give two counterexample to show that both labeled and unlabeled constrained-shape-partitions are not supermodular.

Study of Disk Graphs

Student: Ying-Jen Lin

Advisor: Dr. Ko-Wei Lih

Department of Mathematics

National Taiwan University

Abstract

Disk graphs are motivated to model radio broadcast networks (Hale, 86) and cellular telephone networks. We usually model these networks by an interfere graph with vertices representing transmitters and disks representing their transmitting ranges. And two vertices are adjacent if the signals of transmitters interfere. The problem of assigning distinct frequencies for transmitters with intersecting range corresponds to the minimum coloring problem for disk graphs.

The recognition problems for UD(Breu-Kirkpatrick, 98), ID (Breu-Kirkpatrick-Kratocil, 95) graphs are NP-hard. We find some forbidden subgraphs and examples for UD, ID and CD graphs in this study.

Group Testing Problems

Su-tzu Juan and Gerard J. Chang
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Abstract

This thesis studies group testing problems. The idea of group testing originated from the blood testing in 1942 by Dorfman. Li was the first who studied the combinatorial group testing as follows. Consider a population V of n items consisting of an unknown subset $D \subseteq V$ of d defectives. The problem is to identify the set D by a sequence of group tests. Each test is on a subset X of V with two possible outcomes: a *negative* outcome indicates that $X \cap D = \emptyset$, and a *positive* outcome indicates that $X \cap D \neq \emptyset$. The goal is to minimize the number $M[d,n]$ of tests under the worst scenario.

We may generalize the problem as follows. Consider a population V of n items and a sample space $S \subseteq 2^V$. The problem is to identify an unknown $D \in S$ by a sequence of group tests. Each test is on a subset X of V which partitions S into $S_1 = \{D \in S : D \cap X = \emptyset\}$ and $S_2 = \{D \in S : D \cap X \neq \emptyset\}$. The goal is to identify the unknown D using a minimum number $M[S]$ of tests under the worst scenario.

Chapter 2 considers the case when $G = (V,S)$ is a graph. We also use $M[G]$ for $M[S]$. Damaschke proved that $\lceil \log_2 e(G) \rceil \leq M[G] \leq \lceil \log_2 e(G) \rceil + 1$ for any graph G, where $e(G)$ is the number of edges of G. Chapter 2 gives an improved bound for general graphs G. Namely, if G with $2^{k-1} < e(G) \leq 2^{k-1} + 2^{k-9} + 2^{k-15} + 2^{\frac{k+1}{2}} + 2^{\frac{k-5}{2}} + 2^{\frac{k-7}{2}} + 2^{\frac{k-9}{2}}$ and $k \geq 15$, then $M[G] = \lceil \log_2 e(G) \rceil$. While there are infinitely many complete graphs G with $M[G] = \lceil \log_2 e(G) \rceil + 1$, it was conjectured by Chang and Hwang that $M[G] = \lceil \log_2 e(G) \rceil$ for all bipartite graphs G. Chapter 2 also verifies the conjecture for bipartite graphs G with $e(G) \leq 2^5$ or $2^{k-1} < e(G) \leq 2^{k-1} + 2^{k-3} + 2^{k-4} + 2^{k-5} + 2^{k-6} + 2^{k-7} + 27 \cdot 2^{\frac{k-8}{2}} - 1$ for $k \geq 6$.

Chapter 3 considers group testing on hypergraphs $H = (V,S)$. We also use $M[H]$ for $M[S]$. Triesch proved for any hypergraph $H = (V,E)$ of rank r, it is the case that $M[H] \leq \log_2 e(H) + r - 1$, where $e(H)$ means the number of edges of H. This chapter proves that for any hypertree $H = (V,E)$, we have $M[H] = \lceil \log_2 e(H) \rceil$.

Chapter 4 considers another kind of group testing called the consecutive positive problem. In a set V_n of n items, with linear order $\prec$, each item has an associated *state* positive or negative. The set V_n has the *d-consecutive positive property* if the set of positive items is a consecutive set (under the order $\prec$), and contains at most d items. Chapter 4 studies the problem of finding these consecutive positive items from V_n. Colbourn proved that this can be accomplished with $O(\log_2 d + \log_2 n)$ times for at most d consecutive positives in a linearly ordered n-set of items. Colbourn didn't give the exact value of times needed. Chapter 4 gives the exact values for $d = 1, 2, 3$, and gives an upper bound $\lceil \log_2(nd) \rceil + 6$ for $d \geq 4$.

Density of the Circular Chromatic Numbers of Series-Parallel Graphs

Student : Zhishi Pan Advisor : Xuding Zhu

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National Sun Yat-sen University
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Abstract

Suppose G is a series-paralle graph. It was proved that either $\chi_c(G) = 3$ or $\chi_c(G) \leq 8/3$. So none of the rationals in the interval $(8/3, 3)$ is the circular chromatic number of a series-parallel graph. This paper proves that for every rational $r \in [2, 8/3] \cup \{3\}$ there exist a series-parallel G with $\chi_c(G) = r$.

On the Size of k-forced Critical Set

Student: Shih-Chen Tai

Advisor: Hung-Lin Fu
Chin-Lin Shiue

Department of Applied Mathematics
National Chiao Tung University

Abstract

A critical set is a minimal uniquely completable partial latin square. The size of a critical set is the number of the filled cells within it. In this thesis, we first introduce the notion of k-forced critical set and then study the size of such critical sets.

Broadcasting Problem in Communication Networks

Student : Gen-Wen Tien Advisor : Gerard J. Chang

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Abstract

We define broadcasting from an originator to be the process of passing one unit of information from that source to every other vertex in a connected graph $G = (V, E)$. This is accomplished by a series of calls over the edges of G, subject to the following constraints: (1) each call requires one unit of times; (2) a vertex can only call an adjacent vertex; and (3) a vertex can participate in only one call per unit of time. We mainly discuss the three following problems: (1) finding the broadcast mumber $b(u)$ of a vertex u which is the minimum number if calls needed to broadcast the information to all other vertices; (2)conputing the broadcast number of G, denoted by $b(G)$, which is the minimum broadcast number of a vertex in G; (3) determining the broadcast center of G, denoted by $\mathrm{BC}(G)$, which is the set of all vertices having minimum broadcast number. In this thesis, we simplify Slater, Cockayne and Hedetniemi's algorithm for finding the broadcast center $\mathrm{BC}(T)$ of a tree T by using a labeling algorithm. We also study the problem of broadcasting messages with multiple originators for paths, cycles, full m-ary trees, complete graphs, and complete bipartite graphs.

Double-Loop Networks with Minimum Diameter

Student: C. P. Wang Advisor: Chiuyuan Chen

Department of Applied Mathematics
National Chiao Tung University

Abstract

Double-loop networks have been widely studied as an architecture for local area networks. Let N denote the number of stations in a double-loop network and let $d(N)$ be the best possible diameter of a double-loop network with N vertices. Wong and Coppersmith showed that $d(N) \geq \lceil \sqrt{3N} \rceil - 2$. This is a well-known lower bound for $d(N)$ and is usually denoted as $\ell b(N)$. Given an N, if one can find a double-loop network with its diameter being equal to $\ell b(N)$, then clearly this network is a minimum diameter double-loop network with N stations; this is the way that many authors found minimum diameter double-loop networks for some classes of values of N. When $d(N) > \ell b(N)$, only a few results about minimum diameter double-loop networks are known. In this paper, we find minimum diameter double-loop networks for two classes of values of N when $d(N) = \ell b(N) + 1$.