

# 2001 組合數學新苗 研討會



Jun. 16-17, 2001

主辦單位：淡江大學數學

贊助單位：國科會數學研究推動中心

# 2001 組合數學新苗研討會

JUNE 16

**INVITED SPEECH** 主持人：李國偉

9:00 ~ 10:00 葉鴻國

**SESSION 1** 主持人：顏經和

10:00 ~ 10:25 鍾憲輝

10:25 ~ 10:50 洪加進

10:50 ~ 11:05 tea time break

**SESSION 2** 主持人：阮夙姿

11:05 ~ 11:30 王規樺

11:30 ~ 11:55 莊枏樺

11:55 ~ 12:20 吳瑞瑜

12:10 ~ 13:30 lunch

**SESSION 3** 主持人：董立大

13:30 ~ 13:55 陳建銘

13:55 ~ 14:20 吳姣姣

14:20 ~ 14:45 吳文瑞

14:45 ~ 15:00 tea time break

**SESSION 4** 主持人：葉光清

15:00 ~ 15:25 陳奕仁

15:25 ~ 15:50 潘志實

15:50 ~ 16:15 鍾孟儒

16:15 ~ 16:30 tea time break

**SESSION 5** 主持人：林強

16:30 ~ 16:55 張原禎

16:55 ~ 17:20 李崇道

17:30 ~ dinner

JUNE 17

**INVITED SPEECH** 主持人：張鎮華

9:00 ~ 10:00 陳伯亮

**SESSION 1** 主持人：翁志文

10:00 ~ 10:25 林鴻宇

10:25 ~ 10:50 張欣心

10:50 ~ 11:05 tea time break

**SESSION 2** 主持人：周文賢

11:05 ~ 11:30 范慧蘭

11:30 ~ 11:55 廖崇碩

11:55 ~ 12:20 李珠矽

12:20 ~ 13:30 lunch

**SESSION 3** 主持人：傅恆霖

13:30 ~ 13:55 張吉南

13:55 ~ 14:20 孫一凡

14:20 ~ 14:45 廖啓明

14:45 ~ 15:00 tea time break

**SESSION 4** 主持人：陳哲炯

15:00 ~ 15:25 曾湄菁

15:25 ~ 15:50 林添龍

# 2001 組合數學新苗研討會

JUNE 16

報到時間：8:30AM~9:00AM

**INVITED SPEECH** 主持人：李國偉 教授（中央研究院）

9:00 ~ 10:00 葉鴻國 教授（高雄大學應用數學系）  
*A guided tour of concentration inequalities*

**SESSION 1** 主持人：顏經和 教授（真理大學）

10:00 ~ 10:25 鍾憲輝（淡江大學數學系）  
*The study of decomposing  $2K_{p,q,r,s}$  into most cycles*

10:25 ~ 10:50 洪加進（東吳大學數學系）  
*Extended 5-cycle Systems*

10:50 ~ 11:05 **tea time break**

**SESSION 2** 主持人：阮夙姿 教授（暨南大學）

11:05 ~ 11:30 王規樺（東吳大學數學系）  
*Mendelsohn Triple System with repeated elements in blocks*

11:30 ~ 11:55 莊柎樺（淡江大學數學系）  
*The study of decomposing  $\lambda K_{m(n)}$  into most cycles.*

11:55 ~ 12:20 吳瑞瑜（暨南大學資工所）  
*Hamiltonicity of pyramid networks*

12:10 ~ 13:30 **lunch**

**SESSION 3** 主持人：董立大 教授（東海大學）

13:30 ~ 13:55 陳建銘（東海大學數學系）  
*The Hamiltonicity of Multiloop Networks*

13:55 ~ 14:20 吳姣姣（中山大學應用數學系）  
*Game chromatic number of Halin graphs*

14:20 ~ 14:45 吳文瑞（成功大學數學系）  
*Automorphism groups of certain  $2-(v,3, \lambda)$  designs from finite fields*

14:45 ~ 15:00 **tea time break**

# 2001 組合數學新苗研討會

## SESSION 4 主持人：葉光清 教授（逢甲大學）

- 15:00 ~ 15:25 陳奕仁（逢甲大學應用數學系）  
*A Study on Distance-Dependent Labelings of Graphs*
- 15:25 ~ 15:50 潘志實（中山大學應用數學系）  
*Circular chromatic number of series-parallel graphs of large odd girth*
- 15:50 ~ 16:15 鍾孟儒（淡江大學數學系）  
*The study of the perfect map.*
- 16:15 ~ 16:30 **tea time break**

## SESSION 5 主持人：林強 教授（中央大學）

- 16:30 ~ 16:55 張原禎（中央大學數學系）  
*Medians and Total Displacements of Weighted Graphs*
- 16:55 ~ 17:20 李崇道（義守大學應用數學系）  
*The Implementation of Binary (89,45,17) Quadratic Residue Code up to five errors*
- 17:30 ~ **dinner**

## JUNE 17

報到時間：**8:30AM~9:00AM**

## INVITED SPEECH 主持人：張鎮華 教授（交通大學）

- 9:00 ~ 10:00 陳伯亮 教授（台中技術學院）  
*The study of near automorphism of graphs*

## SESSION 1 主持人：翁志文 教授（交通大學）

- 10:00 ~ 10:25 林鴻宇（交通大學應用數學系）  
*Matrix transformation networks*
- 10:25 ~ 10:50 張欣心（交通大學應用數學所）  
*Some Result for Double-Loop Networks*
- 10:50 ~ 11:05 **tea time break**

# 2001 組合數學新苗研討會

## SESSION 2 主持人：周文賢 教授（中央研究院）

- 11:05 ~ 11:30 范慧蘭（交通大學應用數學系）  
*The Edge-Colorings of Graphs with Small Genus*（低虧格圖的邊著色）
- 11:30 ~ 11:55 廖崇碩（交通大學應用數學系）  
*k-Tuple domination in graphs*
- 11:55 ~ 12:20 李珠矽（交通大學應用數學系）  
*Permutation Polytopes and Optimal Partition*
- 12:20 ~ 13:30 **lunch**

## SESSION 3 主持人：傅恆霖 教授（交通大學）

- 13:30 ~ 13:55 張吉南（真理大學數學系）  
*A Study on graceful labeling of disconnect graphs*
- 13:55 ~ 14:20 孫一凡（交通大學應用數學系）  
*Book-Embeddings in Graphs*（圖形的書式嵌入）
- 14:20 ~ 14:45 廖啓明（交通大學應用數學系）  
*On the Ramsey Number of  $R(mP_n, mP_n)$*
- 14:45 ~ 15:00 **tea time break**

## SESSION 4 主持人：陳哲炯 教授（中國技術學院）

- 15:00 ~ 15:25 曾湄菁（交通大學應用數學系）  
 *$T_k$ -free Domination in Graphs*
- 15:25 ~ 15:50 林添龍（淡江大學數學系）  
*4 cycle system in  $K_{2m, 2n}$  with 2-regular leaves.*

# A guided tour of concentration inequalities

葉鴻國 教授

高雄大學應用數學系

## Abstract

We give a basic introduction to the topic of concentration bounds, typically with exponentially small error probabilities, for random variables occurring in discrete probability. We survey some of the main methods for proving such inequalities and give a few examples to the way these estimates are used. This talk aims at making known results and their proofs accessible to a wider audience.

McDiarmid

"Concentration"

probability methods for Algorithmic Dis Met'

# The study of decomposing $2K_{p,q,r,s}$ into most cycles

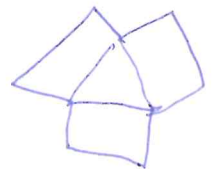
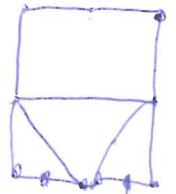
Student: 鍾憲輝      Advisor: 高金美

淡江大學數學系

## Abstract

Let  $2K_{p,q,r,s}$  denote the 2-fold complete four partite graph of order  $p, q, r, s$  with the partite sets  $\{a_1, a_2, \dots, a_p\}$ ,  $\{b_1, b_2, \dots, b_q\}$ ,  $\{c_1, c_2, \dots, c_r\}$ , and  $\{d_1, d_2, \dots, d_s\}$ . Any two distinct vertices are joined with two edges if and only if they belong to different parts.

To decompose  $2K_{p,q,r,s}$  into most cycles is the same as to decompose  $2K_{p,q,r,s}$  into as small cycle as possible. So, we try to decompose  $2K_{p,q,r,s}$  into as many triangles as possible. In this paper, we will consider three cases of  $2K_{p,q,r,s}$ :  $p = q = r = s$ ,  $p = q = r \neq s$ ,  $p = q \neq r = s$ , for positive integers  $p, q, r, s$ . We show that how to decompose  $2K_{p,q,r,s}$  into most triangles and most cycles.



# Extended 5-cycle Systems

Student: 洪加進      Advisor: 黃文中

東吳大學數學系

## Abstract

An extended 5-cycle system of order  $n$  ( $E5CS(n)$ ) is a pair  $(S, E)$  where  $S$  is an  $n$ -set and  $E$  is a collection of shift 5-tuple (called them blocks) of type  $(a, b, c, d, e)$ ,  $(a, a, b, c, b)$  and  $(a, a, a, a, a)$ , such that each unordered pair (not necessarily distinct) belongs to exactly one block. In graph notation, An  $E5CS(n)$  is equivalent to the decomposition of the edges of  $K_n^l$  into 5-cycle, tadpoles and loops. In this talk, we will show that  $E5CS(n)$  exists for all  $n$  except  $n = 2$  and 3.



# Mendelsohn Triple System with repeated elements in Blocks

Student: 王規樺      Advisor: 黃文中

東吳大學數學系

## Abstract

An extended Mendelsohn triple system of order  $v$  is a collection of cyclic triples of type  $[x, y, z]$ ,  $[x, x, y]$  and  $[x, x, x]$ , chosen from a  $v$ -set, such that each ordered pair (not necessarily distinct) belongs to exactly one triple. In graph notation, an extended Mendelsohn triple system of order  $v$  is equivalent to the decomposition of the arcs of  $D_v^+$  into cyclic triples, lollipops and loops. The digraph  $\lambda D_v^{+\alpha}$  is the graph obtained by attaching  $\alpha$  loops to each vertex of  $\lambda D_v^+$ . In this talk, we consider the decomposition of digraph  $\lambda D_v^{+\alpha}$ , where  $\alpha = 1, 2$  or  $3$ , into cyclic triples, lollipops and loop-free.

# The study of decomposing $\lambda K_{m(n)}$ into most cycles

Student: 莊柵樺    Advisor: 高金美

淡江大學數學系

## Abstract

A packing of  $G$  with triangles is an ordered triple  $(S, H, L)$ , where  $S$  is the vertex set of  $G$ .  $H$  is a collection of edge disjoint triangles of the edge set of  $G$  and  $L$  is the set of edges in  $G$  not belonging to a triangle of  $H$ . the set of edge in  $L$  is called the leave.  $\lambda G$  means each edge of  $G$  has to be used  $\lambda$  times.

If  $|H|$  is as large as possible, or equivalently  $|L|$  is as small as possible then the packing is said to be maximum, and  $L$  is a minimum leave.

In this paper, we discuss the maximum packing and minimum leave of  $\lambda K_n$  with triangles for  $\lambda > 1$ ,  $n \geq 3$ ,  $\lambda, n$  are integers.

# Hamiltonicity of pyramid networks

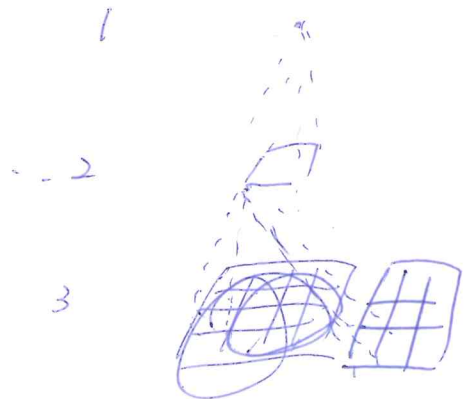
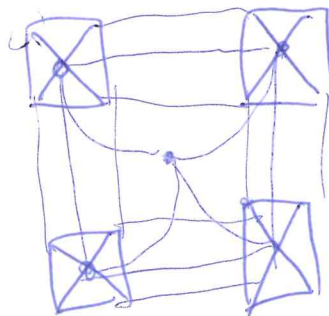
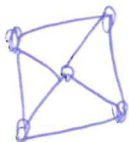
Student: 吳瑞瑜      Advisor: 杜迪榕

暨南大學資工所

## Abstract

The pyramid network is one of the important architectures in parallel computing, network computing, computer vision, and image processing. Some topological properties such as diameter, connectivity, and fault diameter of pyramid networks have been investigated in literatures. Their results show that pyramid networks have very good fault tolerance properties. In fact, a cycle that contains every node of a network is called a hamitonian cycle. A  $n$ -node network has a hamitonian cycle means that it can embed a ring of length  $n$  with dilation 1 and congestion 1. In this thesis, we study the problem of finding a Hamiltonian cycle and a Hamiltonian cycle with a faulty node or link in pyramid networks. We show how to construct a hamiltonian path between any two nodes in a pyramid network. Then, we prove that a pyramid network is hamiltonian connected. Furthermore, we also construct cycles of arbitrary length in pyramid networks. We show that a pyramid network is pancyclic. Besides, we can also construct all possible even length of cycle in  $n \times n$  mesh when  $n$  is even.

$n=1$



$k$

$P_{2^{k-1}} \square P_{2^{k-1}}$

# The Hamiltonicity of Multiloop Networks

Student: 陳建銘      Advisor: 董立大

東海大學數學系

## Abstract

A multi-loop network  $D_n(h_1, h_2, \dots, h_m)$  is a digraph with  $n$  vertices  $0, 1, \dots, n-1$  and the arc-set  $\{(i, j) : i \equiv j + h_k \pmod{n} \text{ for some } k \in \{1, 2, \dots, m\}\}$ . The multi-loop networks have been widely studied as architecture for local area networks in the last few years. There are many interconnection networks included in multi-loop networks. The existence of a hamilton cycle is one of the most important measurements for interconnection networks. If  $m = 2$  then  $D_n(h_1, h_2)$  is called a double-loop network. Hwang and Li obtained the necessary and sufficient condition for the hamiltonicity of  $D_n(h_1, h_2)$ . In this thesis, we study the hamiltonicity of the digraph  $D_n(h_1, h_2, \dots, h_m)$  for  $m \geq 2$ .

# Game chromatic number of Halin graphs

Student: 吳姣姣      Advisor: 朱緒鼎

中山大學應用數學系

Abstract

# Automorphism groups of certain $2-(v, 3, \lambda)$ designs from finite fields

Student: 吳文瑞      Advisor: 柯文峰

成功大學數學系

## Abstract

Let  $F$  be a finite field of characteristic not 2, and  $S \subseteq F$  a subset with three elements. Consider the collection

$$\mathbf{S} = \{S \cdot a + b \mid a, b \in F, a \neq 0\}.$$

Then  $(F, \mathbf{S})$  is a 2-design, whose blocks may be interpreted as segments from a geometric point of view; and the parameter  $\lambda$  of  $(F, \mathbf{S})$  is 1, 2, 3 or 6. We find in this paper the full automorphism group of  $(F, \mathbf{S})$ . Namely, if we put  $U = \{r \mid \{0, 1, r\} \in \mathbf{S}\}$  and  $K$  the subfield of  $F$  generated by  $U$ , then the automorphisms of  $(F, \mathbf{S})$  are the maps of the form  $x \mapsto g(\alpha(x)) + b$ ,  $x \in F$ , where  $b \in F$ ,  $\alpha : F \rightarrow F$  is a field automorphism fixing  $U$ , and  $g$  is a linear function of  $F$  considered as a vector space over  $K$ .

# A Study on Distance-Dependent Labelings of Graphs

Student: 陳奕仁      Advisor: 葉光清

逢甲大學應用數學系

## Abstract

For positive integers  $d_1, d_2$ , an  $L(d_1, d_2)$ -labeling of a graph  $G$ , is a function  $f : V(G) \rightarrow \{0, 1, 2, \dots\}$  such that  $|f(u) - f(v)| \geq d_i$  whenever the distance between  $u$  and  $v$  is  $i$  in  $G$ , for  $i = 1, 2$ . The  $L(d_1, d_2)$ -number of  $G$ ,  $\lambda_{d_1, d_2}(G)$  is the smallest  $k$  such that there exists an  $L(d_1, d_2)$ -labelling with labels no more than  $k$ . This is a graph labeling (or coloring) with constraints depending on the distance between vertices. It is related to the code assignment problem in computer networks.

Base on the  $L(d, 1)$ -labeling, we will consider another problem. Given an  $(d, 1)$ -labeling  $f$  of  $G$ , the  $L(d, 1)$  edge span of  $f$ ,  $\beta_d(G, f)$ , is defined to be  $\max\{|f(u) - f(v)| : \{x, y\} \in E(G)\}$ . The  $L(d, 1)$  edge span of  $G$ ,  $\beta_d(G)$ , is  $\min \beta_d(G, f)$ , where the minimum runs over all  $L(d, 1)$ -labelings of  $G$ .

This article will study the  $L(d, 1)$  edge span on several classes of graphs.

## References

- [1] G. J. Chang, W.-T. Ke, D. Kuo, D. D.-F. Liu and R. K. Yeh, On  $L(d, 1)$ -labelings of graphs, *Discrete Math.* **220** (2000), 57-66.
- [2] J. Georges and D. W. Mauro, Generalized vertex labelings with a condition at distance two, *Congr. Numer.* **109** (1995), 141-159.
- [3] J. Georges and D. W. Mauro, On regular graphs optimally labeled with a condition at distance two, manuscript.
- [4] J. R. Griggs and R. K. Yeh, Labeling graphs with a condition at distance two, *SIAM J. Discrete Math.* **5** (1992), 586-595.
- [5] J. van den Heuvel, R. A. Leese, and M. A. Shepherd, Graph labeling and radio channel assignment, *J. Graph Theory* **29** (1998), 263-283.

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2000 *Mathematics Subject Classification.* 05C12, 05C78, 05C9.

*Key words and phrases.* Graph labeling, distance.

- [6] D. Kuo and J.-H. Yan, On  $L(2, 1)$ -labeling of the Cartesian product of paths and cycles, manuscript.
- [7] D. D.-F. Liu and R. K. Yeh, On distance-two labelings of graphs, *Ars Combin.* 47 (1997), 13-22.
- [8] K.-F. Wu and R. K. Yeh, Labeling graphs with the circular difference, *Taiwanese J. Math.* 4 (2000), 397-405.
- [9] R. K. Yeh, The edge span of distance two labelings of graphs, *Taiwanese J. Math.* 4 (2000), 675-683.



# Circular chromatic number of series-parallel graphs of large odd girth

Student: 潘志實      Advisor: 朱緒鼎

中山大學應用數學系

**Abstract**

# The study of the perfect map

Student: 鍾孟儒      Advisor: 高金美

淡江大學數學系

## Abstract

If  $s$  is a  $c$ -ary cycle of period  $n$ , then we say that  $s$  is a  $v$ -window sequence if no  $c$ -ary  $v$ -tuple occurs in two distinct positions within a period of  $s$ .

A  $c$ -ary de Bruijn sequence of span  $v$  is a  $v$ -window sequence of period equal to  $c^v$ . That is, every possible  $c$ -ary  $v$ -tuple occurs precisely once in a period of a de Bruijn sequence. In this thesis we give a recursive construction to construct  $(2^{tv}, 2^t, v)$  de Bruijn sequence, for given  $v$ .

# Medians and Total Displacements of Weighted Graphs

Student: 張原禎     Advisor: 林強

中央大學數學系

## Abstract

Suppose that  $G$  is a graph with positive weights on edges, i.e, there exists a *weight function*  $w : E(G) \rightarrow R^+$ ;  $w(e)$  is called the *weight* on an edge  $e$ . Then  $(G, w)$  is called a weighted graph.

Suppose that  $(G, w)$  is a connected, weighted graph. For a path  $P$  in  $(G, w)$  the *weight* of  $P$  is defined by

$$w_G(P) = \sum_{e \in E(P)} w(e).$$

For two vertices  $x, y$  in  $G$ , the *weight distance* between  $x$  and  $y$  is defined by  $wd_G(x, y) = \min w_G(P)$ , where the minimum is taken over all paths  $P$  which join  $x$  and  $y$ . For a vertex  $x$  the *weight sum* of  $x$  is

$$ws_G(x) = \sum_{y \in V(G)} wd_G(x, y).$$

The *weight sum* of a graph  $G$  is  $ws(G) = \min_{x \in V(G)} ws_G(x)$ . If a vertex  $v$  satisfies  $ws_G(v) = ws(G)$ , then  $v$  is called a *weight median* of  $(G, w)$ . If  $w(e) = 1$  for every edge  $e$  in  $G$ , then we denote  $ws(G)$  by  $s(G)$ .

Suppose  $\phi$  is a permutation of  $V(G)$ . Then the *weight displacement* of  $\phi$  is defined by

$$wD(\phi) = \sum_{x \in V(G)} wd_G(x, \phi(x)).$$

The *weight displacement* of  $G$  is defined by  $wD(G) = \max wD(\phi)$ , where the maximum is taken over all permutations  $\phi$  of  $V(G)$ .

In this paper, we consider

- (1) The locations of weight medians of a connected, weighted graph.
- (2) The range of  $s(G)$  if  $G$  is a connected graph of order  $n$  and with maximum degree  $k$ .
- (3) The relationship between the weight sum and the weight displacement of a connected graph.

# The Implementation of Binary (89,45,17) Quadratic Residue Code up to five errors

Student: 李崇道      Advisor: 張耀祖

義守大學應用數學系

## Abstract

The main purpose of this thesis is, by applying a new algebraic decoding method used by R. He et al [1] to develop an algorithm for decoding the binary (89, 45, 17) code up to five errors. The original algebraic decoding method is applying Sylvester resultant to reduce the unknowns and to solve the nonlinear multivariate equations in the Newton identities. The new method is use a matrix-determinant technique to produce phantom-relations of unknowns as auxiliary equations. This will add the number of equations that are needed to solve the error-locator polynomial.

It is not necessary to use the new method for the first three errors. We apply the new method to treat the cases of four and five errors. With the help of the software Maple V and Mathematica we can solve complicated equations of multivariate. Base on the result obtained above, a C++ program was developed and was run in a Intel's P-III personal computer to implementing the decoding algorithm. An image was checked successively through our decoding algorithm for the binary (89, 4, 17) QR code up to five errors.

Problem:  $D(G) = 2|V(G)| - 4 \iff G$  is bipartite and  $G$  ? 1989

## The study of near automorphism of graphs

陳伯亮 教授\*

台中技術學院

### Abstract

Let  $f$  denote a permutation of the  $n$  vertices of connected graph  $G$ , and let  $x$  and  $y$  be distinct vertices in  $G$ . Define  $D(f, x, y)$  to be  $|d(x, y) - d(f(x), f(y))|$ , and define  $D(f, G)$  to be sum of  $D(f, x, y)$  over all the  $n!$  unordered pairs  $x, y$  of distinct vertices of  $G$ . Permutation  $f$  (of the vertices of  $G$ ) is an automorphism of  $G$  if and only if  $D(f, G) = 0$ . Let  $D(G)$  denote the smallest positive value of  $D(f, G)$  among the  $n!$  permutations  $f$  of the vertices of  $G$ . A permutation  $f$  for which  $D(G) = D(f, G)$  is called a near-automorphism.

(JCT A 1989)

$$D(P_n) = 2n - 4$$

$$D(K_{m,n}) = \begin{cases} 2(m+n) - 4 & \text{if } m \neq n-1 \\ 2m & \text{if } m = n-1 \end{cases}$$

$$D(K(n_1, n_2, \dots, n_k)) = \begin{cases} \dots \end{cases}$$

$$D(G) \text{ is even (} \because D(f, G) \text{ is even)}$$

$$2 \leq D(G) \leq 2|V(G)| - 4 \text{ if } G \text{ is not complete.}$$

\*Supported in part by the National Science Council under grant NSC88-2914-I-025-001-A1.

Thm:  $D(G) = 2|V(G)| - 4 \iff G$  is bipartite.

$(\dots)$   
 $(\dots)$   
 $(\dots)$

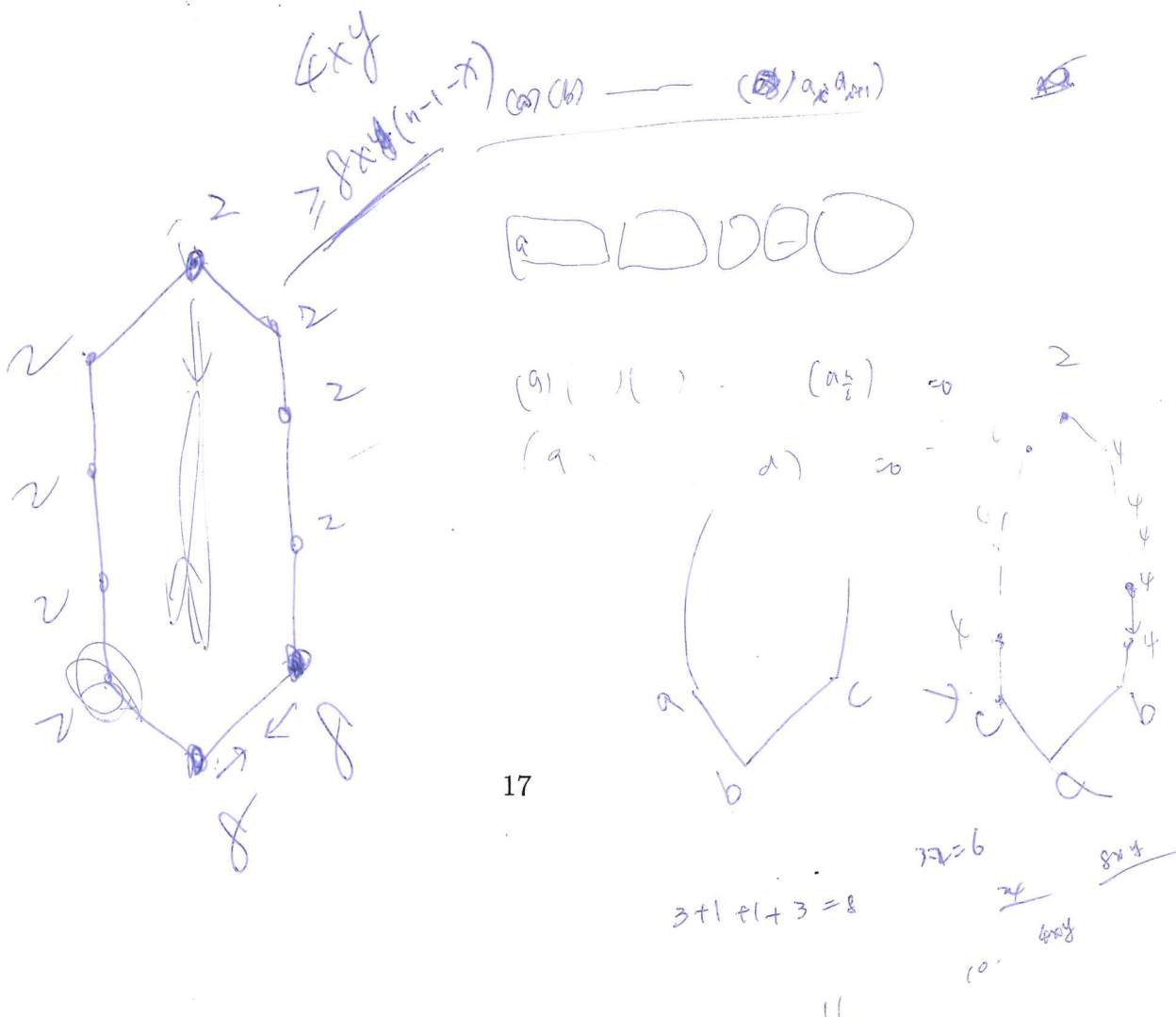
# Matrix Transformation Networks

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## Abstract

In this modern age, multistage interconnection networks become more and more important. One of the main issues is about the nonblocking property of networks. Typical examples are baseline, banyan, and shuffle exchange networks. There are many research on these networks. The most popular topics is to characterize them, since sometimes we waste our time to study the same but look-different network repeatedly. Chang, Hwang and Tong studied bit permutation networks which give these networks a mathematical view, and characterize them by sequences of integers. The attempt of this thesis is to investigate more networks. We discuss the networks in a more general setting by connecting the networks using affine transformation functions.



# Some Result on Double-Loop Networks

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## Abstract

There are two parts of research for double-loop networks of this thesis. First, we make use of program to get optimal diameters for all  $N \leq 5000$  and to get the  $(S1, S2)$  corresponding to  $N$  such that  $S1$  is minimum. From results of program, we draw up a list of  $N$  such that  $d(N) - lb(N) \geq 2$  and a list of  $N$  with  $(S1, S2)$  that  $S1 > 1$ . Farther, we develop the method to find a special family with  $(S1, S2)$  corresponding to it. Second, we make two propositions that number of cells of two  $L$ -shapes will equivalent after growing by conception of procreation.

The Edge-Colorings of Graphs  
with Small Genus  
(低虧格圖的邊著色)

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Abstract



# $k$ -Tuple domination in graphs

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## Abstract

In a graph  $G$ , a vertex is said to dominate itself and all of its neighbors. For a fixed positive integer  $k$ , the  $k$ -tuple domination problem is to find a minimum sized vertex subset in a graph such that every vertex in the graph is dominated by at least  $k$  vertices in this set. The current thesis studies  $k$ -tuple domination in graphs from an algorithmic point of view. In particular, we give linear-time algorithms for the  $k$ -tuple domination problem in trees and strongly chordal graphs by employing a labeling method. We also prove that the  $k$ -tuple domination problem is NP-complete for split graphs (a subclass of chordal graphs) and bipartite graphs.

# Permutation Polytopes and Optimal Partition

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**Abstract**

# A Study on Graceful Labeling of Disconnect Graphs

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## Abstract

Given a graph  $G$ , a graceful labeling  $f$  of  $G$  is an injection from  $f : V \rightarrow \{0, 1, \dots, |E|\}$  such that the function  $f' : E \rightarrow \{1, 2, 3, \dots, |E|\}$ , defined by  $f'(uv) = |f(u) - f(v)|$  for every edge  $uv \in E$ , is a bijection. In this thesis, we study the graceful labeling for the union of two graphs  $G_1, G_2$ , where each  $G_1$  is a path, cycle or  $P_{a,b}$  and  $G_2$  is a path. We also study the graceful labeling for the  $K_n \square S_{2^{n-1} - \binom{n}{2} - 1}$ .

# Book-Embeddings in Graphs

## (圖形的書式嵌入)

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### Abstract

In this thesis we study the three germane measures of the quality of a book-embedding: the thickness (number of pages) of the book, the individual and cumulative widths of the pages, and the number of distinct vertex types. To embed graph  $G$  in a book, with its vertices on the spine of the book and its edges on the pages, in such a way that edges residing on the same page do not cross.

In devising an embedding, One strives to minimize both the number of pages used and the “cutwidth” of the edges on each page. Our main results focus on trees,  $X$ -trees, complete graph and the  $k$ -depth  $K_n$ -cylinder  $C(k, n)$ .

The other works is to minimize the number of distinct vertex types. The type of a vertex  $v$  in a  $p$ -page book-embedding is the  $p \times 2$  matrix of non-negative integers

$$\tau(v) = \begin{pmatrix} l_{v,1} & r_{v,1} \\ \vdots & \vdots \\ l_{v,p} & r_{v,p} \end{pmatrix},$$

where  $l_{v,i}$  (respectively,  $r_{v,i}$ ) is the number of edges incident to  $v$  that connect on page  $i$  to vertices lying to the left (respectively, to the right) of  $v$ . The typenumber of a graph  $G$ ,  $T(G)$ , is the minimum number of different types among all the book-embeddings of  $G$ . We prove that  $T(L_n) = 4$  for  $n \geq 3$ , the typenumber of a tree  $T$  is independent of the number of the pages in a book-embedding, and is equal to either  $|D(T)|$  or  $|D(T)| + 1$  where  $D(T)$  is the set of integers which are degrees of the vertices of  $T$  and then completely characterize trees having typenumber  $|D(T)|$  and trees having typenumber  $|D(T)| + 1$ .

# On the Ramsey Number of $R(mP_n, mP_n)$

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## Abstract

For a fixed graph  $G$ , we define the smallest integer  $r = R(G)$  to be the order of a complete graph  $K_r$  such that no matter how we assign two colors to the edges of  $K_r$ , there exists a monochromatic subgraph which is isomorphic to  $G$ . In this thesis, we show that for  $2 \leq n \leq 7$ ,  $R(mP_n) = m \left( n + \left\lceil \frac{n}{2} \right\rceil \right) - 1$  for any  $m$ .

# $T_k$ -free Domination in Graphs

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## Abstract

The  $T_k$ -free domination number  $\gamma(G; -\mathcal{T}_k)$ ,  $k \geq 2$ , of a graph  $G$ , is the minimum cardinality of a dominating set  $D$  in  $G$  such that the subgraph  $\langle D \rangle$  induced by  $D$  contains no tree of  $k$  vertices as a (not necessarily induced) subgraph; or equivalently, each component of  $\langle D \rangle$  has less than  $k$  vertices. When  $k = 2$ , the  $T_k$ -free domination number is the independent domination number; when  $k \geq (n+1)/2$ , the  $T_k$ -free domination is the usual domination. In this thesis, we study  $T_k$ -free domination from an algorithmic point of view. In particular, we present efficient algorithms for the problem on trees and block graphs by using a dynamic programming method.

# 4 cycle system in $K_{2m,2n}$ with 2-regular leaves

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## Abstract

A regular graph is a graph with the same degree for each vertex. A 2-regular graph is a graph such that the degree of each vertex is 2. A 4-cycle system is a collection of edge-disjoint 4-cycles. In this thesis we prove that there exists a 4-cycle system in the complete bipartite graph with 2-regular leaves by using a recursive construction.