

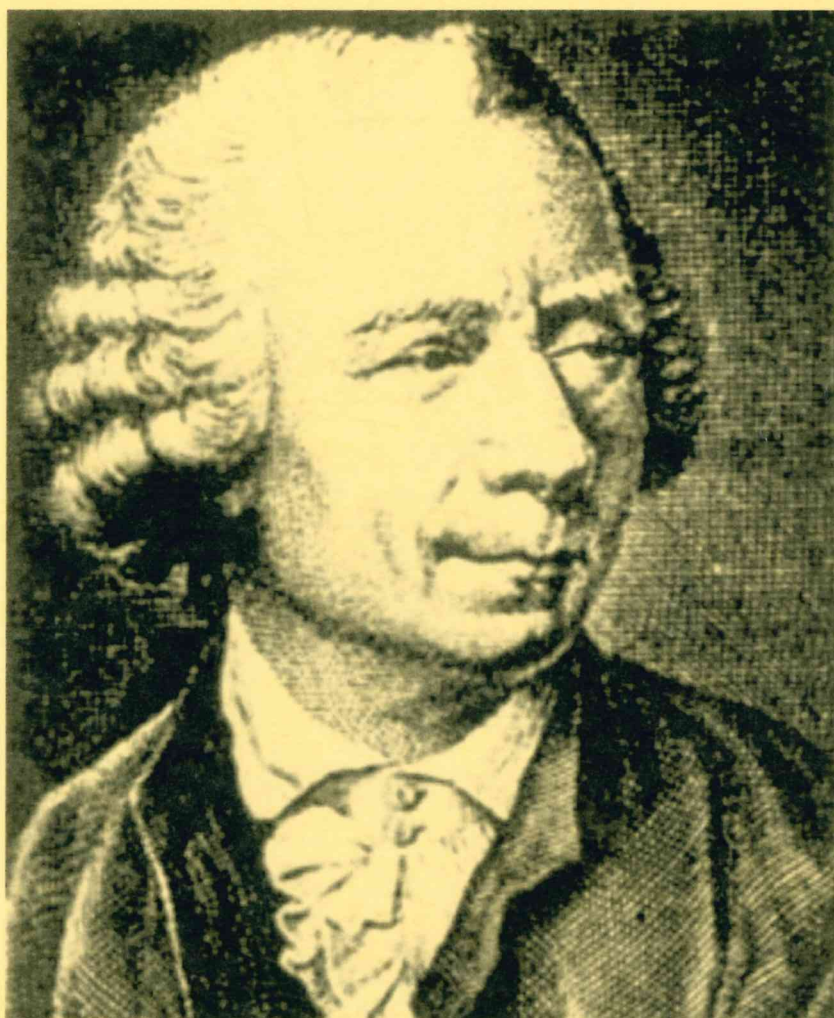
# 2007 年組合數學(新苗)研討會

## 2007 Combinatorics Conference

時間：2007 年 08 月 10 日至 08 月 11 日

地點：高雄市國立中山大學理學院小劇場

網址：<http://mail.math.nsysu.edu.tw/~comb2007/>



Leonhard Euler (1707-1783)

主辦單位：國立中山大學應用數學系

贊助單位：國科會數學研究推動中心

# 2007 組合數學新苗研討會

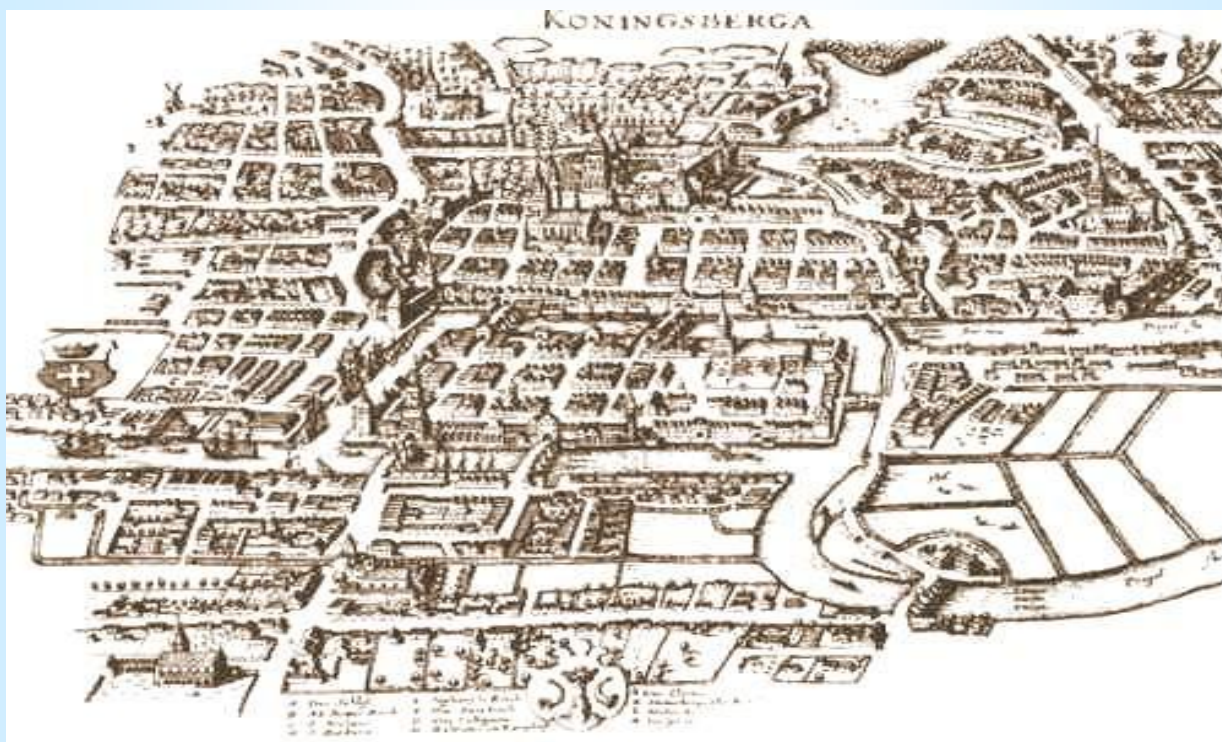
時間：2007.08.10~08.11

地點：高雄市國立中山大學理學院小劇場

網址：[mail.math.nsysu.edu.tw/~comb2007/](mailto:mail.math.nsysu.edu.tw/~comb2007/)

大會主講：張鎮華教授(台灣大學數學系)

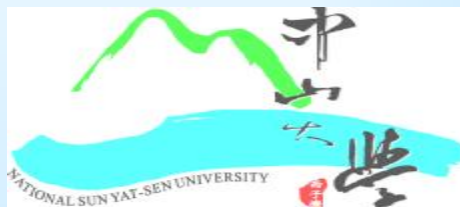
傅恆霖教授(交通大學應用數學系)



主辦單位：國立中山大學應用數學系

贊助單位：國科會數學研究推動中心

籌備委員：朱緒鼎、王彩蓮、董立大



國立中山大學應用數學系

地址：80424 高雄市鼓山區蓮海路 70 號 應用數學系

電話：07-5252000 轉 3832 或 3801~3803 傳真：07-5253809

Email：[comb2007@math.nsysu.edu.tw](mailto:comb2007@math.nsysu.edu.tw)

## 目錄 (Table of Contents)

封面：偉大數學家 Leonhard Euler (出生 300 年紀念)*	
海報(Poster).....	01
目錄 (Table of Contents).....	02
議程 (Conference Program).....	03
演講者名單及其題目與摘要頁次.....	06
演講題目與摘要 (Titles and Abstracts) .....	09
參與人員名單.....	40
地圖 (Maps).....	46

\*在 Graph Theory 或 Discrete Mathematics 的課程裡，應該會接觸到七橋問題 (或 Problem of the Königsberg Bridges) 與平面圖的點邊面公式 (或 Euler's formula)；這兩個重要的古典理論就是由大數學家 Euler 提出來的。Euler 出生於瑞士 Basel，在數學許多領域皆有很大的成就。雖然在他人生的最後 17 年過著眼盲的日子，他仍然持續數學的思考與創作。

2007年組合數學(新苗)研討會議程(Conference Program)

地點: 國立中山大學理學院小劇場

08月10日 星期五

08:00~8:50 報到(Registration)

08:50~9:00 開幕(Welcoming Remarks) 主持人(Chairman): 朱緒鼎教授

**Session 1.** 邀請演講(Invited Talk) 主持人(Chairman): 李國偉教授

09:00~09:50 演講者(Speaker): 張鎮華教授(臺灣大學數學系)

題目(Title): Vertex Coloring Induced by Edge Weighting

09:50~10:00 休息(Tea Break)

**Session 2.** 主持人(Chairman): 葉鴻國教授

10:00~10:25 演講者(Speaker): 蔡馬良(東華大學應用數學系) 博士

題目(Title): The  $L(3,2,1)$ -Labeling of Graphs

10:25~10:50 演講者(Speaker): 張宏鏞(中山大學應用數學系) 博士

題目(Title): Game Colourings of Graphs

✓ 10:50~11:15 演講者(Speaker): 鄧乃心(臺灣大學數學系) 碩士

題目(Title): Vertex Ranking of Graphs

11:15~11:20 休息(Tea Break)

**Session 3.** 主持人(Chairman): 潘俊杰教授

11:20~11:45 演講者(Speaker): 陳宜廷(交通大學應用數學系)

題目(Title): Cyclic Triples

11:45~12:10 演講者(Speaker): 郭志銘(交通大學應用數學系)

題目(Title): Diameters of Generalized De Bruijn Graphs

12:10~13:10 午餐(Lunch)

**Session 4.** 主持人(Chairman): 林 強教授

13:10~13:35 演講者(Speaker): 李張圳(交通大學應用數學系)

題目(Title): Transversals in  $m \times n$  Arrays

\* 13:35~14:00 演講者(Speaker): 陳億庭(中央大學數學系)

題目(Title): 2-decomposable and 3-decomposable Star Forests

14:00~14:25 演講者(Speaker): 葉政峰(中央大學數學系)

題目(Title): On List Coloring of Graphs

14:25~14:35 休息(Tea Break)

**Session 5.** 主持人(Chairman): 郭大衛教授

14:35~15:00 演講者(Speaker): 黃俊瑋(東華大學應用數學系)

題目(Title): The List- $L(2,1)$ -labeling Problem of Digraphs

15:00~15:25 演講者(Speaker): 洪連志(東華大學應用數學系)

題目(Title): The Oriented- $k$ -graceful Labelings of Graphs

15:25~15:50 演講者(Speaker): 賴麗鈴(東華大學應用數學系)

題目(Title): Global Defensive Alliance of Trees and Cartesian Product of Graphs

15:50~16:00 休息(Tea Break)

**Session 6.** 主持人(Chairman): 葉光清教授

16:00~16:25 演講者(Speaker): 廖虹雅(逢甲大學應用數學系)

題目(Title): Graph Labelings with Three Levels of Constraints

16:25~16:50 演講者(Speaker): 宋彥萱(逢甲大學應用數學系)

題目(Title): Distance Labeling on Cartesian Products of Elementary Graphs

16:50~17:15 演講者(Speaker): 楊千慧(逢甲大學應用數學系)

題目(Title): Radio 3-Colorings of Lattice Graphs

17:15~17:40 演講者(Speaker): 沈志文(逢甲大學應用數學系)

題目(Title): A Study on a Magic Labeling Problem

18:30~ 晚宴(Banquet)

## 08月11日 星期六

**Session 7.** 邀請演講(Invited Talk) 主持人(Chairman): 朱緒鼎教授

09:00~09:50 演講者(Speaker): 傅恆霖教授(交通大學應用數學系)

題目(Title): Unsolved Problems in Graph Decompositions

09:50~10:00 休息(Tea Break)

<sup>ss</sup>**Section 8.** 主持人(Chairman): 陳秋媛教授

10:00~10:25 演講者(Speaker): 張雁婷(交通大學應用數學系)

題目(Title): A Study of Berge's Strong Path Partition Conjecture

10:25~10:50 演講者(Speaker): 張樹仁(交通大學應用數學系)

題目(Title): Decomposing the Complete Graph into Paths

10:50~11:15 演講者(Speaker): 李維鴻(淡江大學數學系)

題目(Title): 完全三分圖分割成星狀圖的探討

11:15~11:20 休息(Tea Break)

**Session 9.** 主持人(Chairman): 江南波教授

11:20~11:45 演講者(Speaker): 林耀仁(大同大學應用數學系)

題目(Title): A Note on the Sum-saturability of Trees

11:45~12:10 演講者(Speaker): 陳俊達(大同大學應用數學系)

題目(Title): The Study on Joint Structural Importance in Consecutive-k  
Systems

12:10~13:10 午餐(Lunch)

**Session 10.** 主持人(Chairman): 孫一凡教授

13:10~13:35 演講者(Speaker): 賴欣豪(臺灣大學數學系)

題目(Title): Study into Full Orientable Graphs

13:35~14:00 演講者(Speaker): 陳柏澍(交通大學應用數學系)

題目(Title): Optimal All-to-all Personalized Exchange in Generalized  
Shuffle-Exchange Networks

14:00~14:25 演講者(Speaker): 游喬任(政治大學應用數學系)

題目(Title): The Unique Colorability of a Uniform Mixed Hypergraph

14:25~14:35 休息(Tea Break)

**Session 11.** 主持人(Chairman): 嚴志弘教授

14:35~15:00 演講者(Speaker): 林金龍(嘉義大學資訊工程系)

題目(Title): Finding Hamiltonian Cycle on Perfect Matching of  
[hapo'tresabi] Hypotractable Graphs

15:00~15:25 演講者(Speaker): 陳秉宏(中山大學資訊工程系)

題目(Title): Table Driven Algorithm for Joint Sparse Form

15:25~15:50 演講者(Speaker): 歐俊男(暨南大學資訊工程學系)

題目(Title): A Study on Antipodal Labellings Problems

15:50~16:15 演講者(Speaker): 張凱涵(中央大學數學系)

題目(Title): 2-decomposable, 3-decomposable Multipaths and  
t-decomposable Spiders

16:15~16:20 休息(Tea Break)

16:20~ 頒發優良論文獎

**賦歸(End)**

## 2007組合數學(新苗)研討會演講者名單及摘要頁次

### 邀請演講

姓名	服務單位	E-mail	題目	演講場次 摘要頁次
張鎮華	台灣大學數學系	gjchang@math.ntu.edu.tw	Vertex Coloring Induced by Edge Weighting	週五 Session 1 Page 9
傅恆霖	交通大學應用數學系	hifu@math.nctu.edu.tw	Unsolved Problems in Graph Decompositions	週六 Session 7 Page 11

### 學生演講(依校名、系別、姓名排列)

姓名	服務單位	E-mail	題目	指導教授	演講場次 摘要頁次
林耀仁	大同大學應用數學系	jacky550kimo@yahoo.com.tw	A Note on the Sum-saturability of Trees	江南波	週六 Session 9 Page 12
陳俊達	大同大學應用數學系	u9008015@mail.ttu.edu.tw	The Study on Joint Structural Importance in Consecutive-k Systems	張薰文	週六 Session 9 Page 13
陳秉宏	中山大學資訊工程學系	ud17g@yahoo.com.tw	Table Driven Algorithm for Joint Sparse Form	官大智	週六 Session 11 Page 14
張宏鏞	中山大學應用數學系	gianni.chang@msa.hinet.net	Game Colourings of Graphs	朱緒鼎	週五 Session 2 Page 15
張凱涵	中央大學數學系	942201005@cc.ncu.edu.tw	2-decomposable, 3-decomposable Multipaths and t-decomposable Spiders	林強	週六 Session 11 Page 16

## 2007組合數學(新苗)研討會演講者名單及摘要頁次

姓名	服務單位	E-mail	題目	指導教授	演講場次 摘要頁次
陳憶庭	中央大學數學系	u8013800@cc.ncu.edu.tw	2-decomposable and 3-decomposable Star Forests	林強	週五 Session 4 Page 17
葉政峰	中央大學數學系	942201013@cc.ncu.edu.tw	On List Coloring of Graphs	葉鴻國	週五 Session 4 Page 18
鄧乃心	台灣大學數學系	r94221021@ntu.edu.tw	Vertex Ranking of Graphs	張鎮華	週五 Session 2 Page 19
賴欣豪	台灣大學數學系	f89221010@ntu.edu.tw	Study into Fully Orientable Graphs	李國偉	週六 Session 10 Page 20
李張圳	交通大學應用數學系	guess166296@yahoo.com.tw	Transversals in $m \times n$ Arrays	傅恆霖	週五 Session 4 Page 22
張雁婷	交通大學應用數學系	yanting.chang@gmail.com	A Study of Berge's Strong Path Partition Conjecture	傅恆霖	週六 Session 8 Page 23
張樹仁	交通大學應用數學系	u890224@alumni.nthu.edu.tw	Decomposing the Complete Graph into Paths	傅恆霖	週六 Session 8 Page 24
郭志銘	交通大學應用數學系	jyhmkuo@gmail.com	Diameters of Generalized De Bruijn Graphs	傅恆霖	週五 Session 3 Page 25
陳宜廷	交通大學應用數學系	u890238@alumni.nthu.edu.tw	Cyclic Triples	翁志文	週五 Session 3 Page 26
陳柏樹	交通大學應用數學系	chen.poshu@gmail.com	Optimal All-to-All Personalized Exchange in General Shuffle-Exchange Networks	陳秋媛	週六 Session 10 Page 27
洪連志	東華大學應用數學系	m9311006@em93.ndhu.edu.tw	The Oriented- $k$ -graceful Labelings of Graph	郭大衛	週五 Session 5 Page 28



## 2007組合數學(新苗)研討會演講者名單及摘要頁次

姓名	服務單位	E-mail	題目	指導教授	演講場次 摘要頁次
黃俊瑋	東華大學應用數學系	m9311003@em93.ndhu.edu.tw	The List-L(2,1)-labeling Problem of Digraphs	郭大衛	週五 Session 5 Page 29
蔡馬良	東華大學應用數學系	d9011001@mail.ndhu.edu.tw	The L(3,2,1)-Labeling of Graphs	郭大衛	週五 Session 2 Page 30
賴麗鈴	東華大學應用數學系	m9411008@em94.ndhu.edu.tw	Global Defensive Alliance of Trees and Cartesian Product of Graphs	郭大衛	週五 Session 5 Page 31
游喬任	政治大學應用數學系	94751003@nccu.edu.tw	The Unique Colorability of a Uniform Mixed Hypergraph	張宜武	週六 Session 10 Page 32
李維鴻	淡江大學數學系	leevhome@yahoo.com.tw	完全三分圖分割成星狀圖的探討	高金美	週六 Session 8 Page 33
宋彥萱	逢甲大學應用數學系	micky83y@ms27.hinet.net	Distance Labeling on Cartesian Products of Elementary Graphs	葉光清	週五 Session 6 Page 34
沈志文	逢甲大學應用數學系	yshinbbbb@yahoo.com.tw	A Study on A Magic Labeling Problem	葉光清	週五 Session 6 Page 35
楊千慧	逢甲大學應用數學系	hoby520@hotmail.com	Radio 3-Colorings of Lattice Graphs	葉光清	週五 Session 6 Page 36
廖虹雅	逢甲大學應用數學系	honya1201@yahoo.com.tw	Graph Labelings with Three Levels of Constraints	葉光清	週五 Session 6 Page 37
林金龍	嘉義大學資訊工程系	s0950319@mail.ncyu.edu.tw	Finding Hamiltonian Cycle on Perfect Matching of Hypotractable Graphs	賴泳伶	週六 Session 11 Page 38
歐俊男	暨南大學資訊工程學系	ten.you@msa.hinet.net	A Study on Antipodal Labellings Problems	阮夙姿	週六 Session 11 Page 39

# Vertex Coloring Induced by Edge Weighting

于青林教授提出

Gerard Jennhwa Chang 張鎮華  
 Department of Mathematics,  
 National Taiwan University  
 gjchang@math.ntu.edu.tw

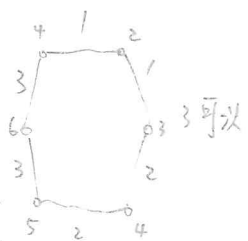
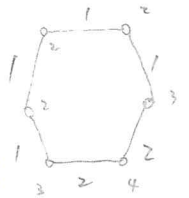
A  $k$ -edge-weighting of a graph is an assignment of an integer weight from the set  $\{1, 2, \dots, k\}$  to each edge. An edge-weighting is *vertex-coloring* (respectively, *vertex-injective*) if for any edge  $uv$  (respectively, every pair of distinct vertices  $u$  and  $v$ ), the colors  $c_u$  and  $c_v$  are distinct, where the color of a vertex is defined as the sum of the weights on the edges incident to that vertex. Karoński, Luczak and Thomason [8] asked if a connected graph of at least three vertices permits a vertex-coloring 3-edge-weighting. They gave a positive answer when the graph is 3-colorable or when a set of real weights is allowed. Addario-Berry et al. [2] showed that a connected graph of at least three vertices permits a vertex-coloring 30-edge-weighting. Minimum  $k$  for which a graph is  $k$ -vertex-injective is called the em irregularity strength of a graph (cf. [7]).

Conjecture

An edge-weighting is *adjacent vertex-distinguishing* (respectively, *vertex-distinguishing*) if for any edge  $uv$  (respectively, every pair of distinct vertices  $u$  and  $v$ ), the multiset of weights appearing on edges incident to  $u$  is distinct from the multiset of weights appearing on the edges incident to  $v$ . Adjacent vertex-distinguishing edge-weighting and vertex-distinguishing edge-weighting have been studied by many researchers [3, 4, 5, 6]. It is clear that a vertex-coloring (respectively, vertex-injective) edge-weighting is adjacent vertex-distinguishing (respectively, vertex-distinguishing), but the converse is not necessarily true. Karoński, Luczak and Thomason [8] proved that every graph without an edge component permits an adjacent edge-distinguishing 213-edge-weighting and that graphs with minimum degree at least  $10^{99}$  permit an adjacent vertex-distinguishing 30-edge-weighting. Addario-Berry et al. [1] improved the results to that every graph without an edge component permits an adjacent edge-distinguishing 4-edge-weighting and that graphs with minimum degree at least 1000 permit an adjacent vertex-distinguishing 3-edge-weighting.

Even people are still not able to verify that a connected graph of at least three vertices permits a vertex-coloring 3-edge-weighting, many graphs are in fact permit vertex-coloring 2-edge-weighting. The current paper is devoted to study bipartite graphs with such property. In particular, we prove that

$C_6$  has no vertex-coloring 2-edge weighting  
 X 2 不行



$P_6$  has a vertex-coloring 2-edge weighting

An edge-weighting is vertex-injective if  $c_u \neq c_v$

for any two distinct vertices  $u$  and  $v$ .

全部不同

a  $k$ -regular bipartite graph with  $k \geq 3$  permits a vertex-coloring 3-edge-weighting. (confirm by (Chang, Lu, Wu and Yu))

a connected bipartite graph  $G$  of at least three vertices permits a vertex-coloring 2-edge-weighting if one of the following property hold: (1)  $G$  has a bipartition set of even size; (2)  $\delta(G) \geq 3$  and  $d(u) \neq \lceil d(v)/2 \rceil$  for any edge  $uv$ ; (3)  $\delta(G) = 1$ . While there are example of connected bipartite graphs permit no vertex-coloring 2-edge-weighting, it remains open to characterize such kind of graphs.

For a bipartite graph  $G$ , someone prove  $\geq 3$  is enough.

## References

- [1] L. Addario-Berry, R. E. L. Aldred, K. Dalal and B. A. Reed, *Vertex colouring edge partitions*, J. Combinatorial Theory B 94 (2005), 237-244.
- [2] L. Addario-Berry, K. Dalal, C. McDiarmid, B. A. Reed and A. Thomason, *Vertex-colouring edge-weightings*, Combinatorica 27 (2007), 1-12.
- [3] M. Aigner, E. Triesch and Zs. Tuza, *Irregular assignments and vertex-distinguishing edge-colorings of graphs*, Combinatorics 90, (A. Barlotti et al., editors), Elsevier Science Pub., New York (1992) 1-9.
- [4] , A. C. Burriss and R. H. Schelp, *Vertex-distinguishing proper edge colorings of graphs*, J. Graph Theory 26 (1997) 73-82.
- [5] P. N. Balister, O. M. Riordan and R. H. Schelp, *Vertex-distinguishing edge colorings of graphs*, J. Graph Theory 42 (2003) 95-109.
- [6] K. Edwards, *The harmonious chromatic number of bounded degree graphs*, J. London Math. Soc. 2 55 (1997) 435-447.
- [7] A. Frieze, R. J. Gould, M. Karoński, F. Pfender, *On graph irregularity strength*, J. Graph Theory 41 (2002) 120-137.
- [8] M. Karoński, T. Luczak and A. Thomason, *Edge weights and vertex colours*, J. Combinatorial Theory B 91 (2004), 151-157.

$C_{4k+2}$  needs 3

$C_{8k+2} + e$  needs 3

Among all vertices of  $G$ , we choose the vertex  $v$  such that ...

# Unsolved Problems in Graph Decompositions

Hung-Lin Fu 傅恆霖

Department of Applied Mathematics,

National Chiao Tung University

hlfu@math.nctu.edu.tw

Let  $G$  be a graph and  $\mathcal{H}$  be a collection of graphs. Then, we say that  $G$  has an  $\mathcal{H}$ -decomposition if the edge set of  $G$  can be partitioned into subsets such that each of them induces a graph in  $\mathcal{H}$ . If  $\mathcal{H}$  contains exactly one graph  $H$ , then we say that  $G$  has an  $H$ -decomposition, denoted by  $H|G$ .

The most well-known problem in graph decomposition is to determine whether a  $\lambda$ -fold complete graph  $\lambda K_v$  has a  $K_k$ -decomposition whenever the following conditions hold: (1)  $v \geq k$ , (2) both  $\lambda(v-1)/(k-1)$  and  $\lambda v(v-1)/k(k-1)$  are positive integers. This is equivalent to the existence of a balanced incomplete block design (BIBD), a  $2-(v, k, \lambda)$  design. Note that the existence of a group divisible design ( $k$ -GDD) can also be considered as a  $K_k$ -decomposition of the balanced complete multipartite graph.

Besides those decompositions close related to design theory, there are quite a few problems in graph theory remained unsolved. In this talk, some of them will be mentioned. Mainly, we start with a problem and report the progress of tackling the problem. For references, several techniques which have been utilized will be included in this talk.

# A Note on the Sum-saturability of Trees

Yao-Ren Lin 林耀仁

Department of Applied Mathematics,

Tatung University

jacky5500kimo@yahoo.com.tw

ADVISOR: Nam-Po Chiang 江南波

Let  $G = (V, E)$  be a graph of order  $n$  and  $s$  be the integer  $n(n+1)/2$ . A 1-1 mapping  $f$  from  $V$  to  $\{1, 2, \dots, n\}$  is a saturating labelling of  $G$  if for each integer  $r \in [1, s]$  there is a connected subgraph  $H$  of  $G$  such that  $\sum_{x \in V(H)} f(x) = r$ . If a saturating labeling of  $G$  exists, then we say  $G$  is sum-saturable. In this note, we determine thoroughly the sum-saturability of all trees of order  $n \leq 7$ . And also we find that all perfect complete  $n$ -ary trees are sum-saturable.

Keywords: Sum-saturable, perfect complete  $n$ -ary tree.

# The Study on Joint Structural Importance in Consecutive- $k$ Systems

Jun-Da Chen 陳俊達

Department of Applied Mathematics,

Tatung University

u9008015@mail.ttu.edu.tw

ADVISOR: Hsun-Wen Chang 張薰文

The *joint structural importance* (JSI) is an important measure of how two components interact in contributing to the system reliability. The value of JSI is positive (negative) if and only if one component becomes more important (less important) when the other works. A *consecutive- $k$ -out-of- $n$  system* is a linear arrangement of  $n$  components such that the system is failed if and only if some consecutive  $k$  components are all failed. In this thesis, we study joint structural importance in the consecutive- $k$ -out-of- $n$  system.

We first completely solve  $JSI(i, j)$  for  $k = 1$  (the series system),  $k = n$  (the parallel system),  $k = n - 1$ , and  $k = n - 2$ , respectively. For the other  $k$ ,  $JSI(1, j') = JSI(1, k) < 0 < JSI(1, n) = JSI(1, j + 2) < JSI(1, j) < JSI(1, k + 1)$ , for  $2 \leq j' \leq k - 1$  and  $k + 3 \leq j \leq n - 1$ . Given a fixed  $i$ , we prove that the  $JSI(i, j)$  is decreasing for  $i - k \leq j \leq i - 1$  and for  $n - k + 1 \leq j \leq n$ , and is nondecreasing for  $i + 1 \leq j \leq i + k$ . Hence we state the graph of  $JSI(i, j)$  has a W-shape property for  $i - k - 1 \leq j \leq i + k + 1$  with  $JSI(i, i) = 0$ . Furthermore, we show that  $JSI(1, j) = JSI(1, n + k + 2 - j)$ . We also present exact formula for several  $JSI(i, j)$  and obtain many comparisons among them.

On the other hand, we present further results on joint structural importance in the consecutive-2 system. We prove that  $JSI(i, i + k) > JSI(i, i + 2k) > JSI(i, i + 3k)$  and  $JSI(i, i + k) > JSI(i, j) > JSI(i, n)$  for  $j \geq i + 3$ . We also present results on half-line case and on combinatorial case. Finally, we make a comparison between structural importance and joint structural importance.

Keywords: Birnbaum importance, joint structural importance, consecutive- $k$  system.

# Table Driven Algorithm for Joint Sparse Form

陳秉宏

Department of Computer Science and Engineering,  
National Sun Yat-Sen University  
m933040040@student.nsysu.edu.tw

ADVISOR: D. J. Guan 官大智

在密碼學的電子簽章系統中，多項相乘計算(Multi-computations)是最重要也是最耗時的計算。該問題始見於Shamir 計算方法，爾後研究便是建立在此基礎上加速運算。其中，有號二進位表示重編法(Binary signed-digit representation recoding algorithm) 中的稀疏格式(Sparse Form), DJM 查表式編碼，聯合稀疏格式(Joint Sparse Form)等等有效降低非零位元的數量。而另一種便是預先計算建表法(Pre-computing algorithm)先將部分結果預算起來並儲存，在藉由移位或者部份碼一起計算來減少計算的次數。所以增加運算速度是建立在減少運算數量。在這篇論文中，DJM 查表編碼是以查表的方式編碼，所以該方法簡單易懂。對聯合稀疏格式而言，建立一個簡單查表編碼方法來完成聯合稀疏格式以及推廣，使得聯合稀疏格式更簡單，更方便人去研究與探討，便是我們的主要撰寫論文的目的。

# Game Colourings of Graphs

Hungyung Chang 張宏鏞

Department of Applied Mathematics,

National Sun Yat-Sen University

gianni.chang@msa.hinet.net

ADVISOR: Xuding Zhu 朱緒鼎

Suppose  $f$  is a graph function which assigns to each graph  $H$  a positive integer  $f(H) \leq |V(H)|$ . An  $f$ -colouring of  $G$  is a mapping  $c : V(G) \rightarrow N$  such that every subgraph  $H$  of  $G$  receives at least  $f(H)$  colours, i.e.,  $|c(H)| \geq f(H)$ . The  $f$ -chromatic number,  $\chi(f, G)$ , is the minimum number of colours used in an  $f$ -colouring of  $G$ . This thesis studies the game version of  $f$ -colouring of graphs. Suppose  $G$  is a graph and  $X$  is a set of colours. Two players, Alice and Bob, take turns colour the vertices of  $G$  with colours from the set  $X$ . A partial colouring of  $G$  is legal (with respect to graph function  $f$ ) if for any subgraph  $H$  of  $G$ , the sum of the number of colours used in  $H$  and the number of uncoloured vertices of  $H$  is at least  $f(H)$ . Both Alice and Bob must colour legally (i.e., the partial colouring produced needs to be legal). The game ends if either all the vertices are coloured or there are uncoloured vertices but there is no legal colour for any of the uncoloured vertices. In the former case, Alice wins the game. In the latter case, Bob wins the game. The  $f$ -game chromatic number of  $G$ ,  $\chi_g(f, G)$ , is the least number of colours that the colour set  $X$  needs to contain so that Alice has a winning strategy. The  $f$ -game chromatic index of a graph  $G$ ,  $\chi'_g(f, G)$ , is the  $f$ -game chromatic number of its line graph. This thesis studies the  $f$ -game chromatic number for the case that  $f$  is one of the following graph functions:

1.  $\text{Rel}_d$  defined as  $\text{Rel}_d(K_{1,d+1}) = 2$  and  $\text{Rel}_d(H) = 1$  otherwise.
2.  $\text{Acy}$  defined as  $\text{Acy}(K_2) = 2$  and  $\text{Acy}(C_n) = 3$  for any  $n \geq 3$  and  $\text{Acy}(H) = 1$  otherwise.
3.  $\text{Path}_i$  defined as  $\text{Path}_i(K_2) = 1$  and  $\text{Path}_i(P_i) = 3$  and  $\text{Path}_i(H) = 1$  otherwise, where  $P_i$  is the path on  $i$  vertices.

In this thesis, we prove that for any outerplanar graph  $G$ ,  $\chi_g(\text{Acy}, G) \leq 7$ . If  $i \geq 8$ , then for any tree  $T$ ,  $\chi_g(\text{Path}_i, T) \leq 9$ . On the other hand, if  $i \leq 6$ , then for any integer  $k$ , there is a tree  $T$  such that  $\chi_g(\text{Path}_i, T) \geq k$ . Suppose  $d \geq 2k^2 + 5k - 1$  and  $G$  is  $k$ -degenerate, then we have  $\chi'_g(\text{Rel}_d, G) \leq 2k + \frac{(\Delta(G)+k-1)(k+1)}{d-2k^2-4k+2}$ .



## 2-decomposable, 3-decomposable Multipaths and $t$ -decomposable Spiders

Kai-han Chang 張凱涵  
Department of Mathematics,  
National Central University  
942201005@cc.ncu.edu.tw

ADVISOR: Chiang Lin 林強

A graph  $G$  is  $t$ -decomposable if and only if  $G$  can be decomposed into  $t$  isomorphic subgraphs. A multipath is a path with multiple edges allowed. A spider is a tree which has a unique vertex with degree  $\geq 3$ . In this thesis, we investigate 2-decomposable and 3-decomposable multipaths and  $t$ -decomposable spiders.

Keywords: decomposition, multipaths, spiders.

## 2-decomposable and 3-decomposable Star Forests

陳億庭

Department of Mathematics,  
National Central University  
u8013800@cc.ncu.edu.tw

ADVISOR: Chiang Lin 林強

A graph  $G$  is  $n$ -decomposable if  $G$  can be decomposed into  $n$  isomorphic subgraphs. A star forest is a forest of which each component is a star. In this thesis, we will investigate 2-decomposable and 3-decomposable star forests.

Keywords: decomposition, star forest.

## On List Coloring of Graphs

Cheng-Feng Yeh 葉政峰  
Department of Mathematics,  
National Central University  
942201022@cc.ncu.edu.tw

ADVISOR: Hong-Gwa Yeh 葉鴻國

In this paper we present some results on list coloring and its variants. A Nordhaus-Gaddum type result on choosability with separation  $s$  is presented which generalizes a theorem of Erdős, Rubin and Taylor (Congr. Numer. **26** (1979) 125-157). A new graph parameter  $\text{ch}_s^g(G)$  is introduced, and its nontrivial upper bound is provided which generalizes a theorem of Waters (J. London Math. Soc. **73** (2006) 565-585). In (Discrete Applied Math. **45** (1993), 277-289.), Tesman showed that if  $P_n$  is a path of  $n$  vertices then  $\text{ch}_s(P_n) = \lfloor 2s(1 - \frac{1}{n}) \rfloor + 1$ . He also remarked that almost the same proof can be easily extended to prove that  $\text{ch}_s(T_n) = \lfloor 2s(1 - \frac{1}{n}) \rfloor + 1$  for a tree  $T_n$  of  $n$  vertices. Here we give a much shorter and neater proof for Tesman's result on  $\text{ch}_s(P_n)$  (and hence also on  $\text{ch}_s(T_n)$ ). In (Discrete Appl. Math. **82** (1998) 1-13) Alon and Zaks proved that  $\text{ch}_s(K_{n,n}) = O(s \log n)$ . In this paper we present a slightly stronger version of their result. For any finite graph  $G$ , Waters (J. London Math. Soc. **73** (2006) 565-585) showed that  $\lim_{s \rightarrow \infty} \text{cch}_s(G)/s$  exists, and define this limit as  $\tau(G)$ . In the last part of this paper, we show that there is another characterization of  $\tau(G)$ ,  $\tau(G) = \inf_{s \in \mathbb{N}} \text{cch}_s(G)/s$ .

Keywords: list coloring.

# Vertex Ranking of Graphs

Nancy N.H. Teng 鄧乃心  
Department of Mathematics,  
National Taiwan University  
r94221021@ntu.edu.tw

ADVISOR: Gerard Jennhwa Chang 張鎮華

Vertex ranking of a graph  $G$  is a mapping  $f$  from  $V(G)$  to the set of all natural numbers such that for any path between two distinct vertices  $u$  and  $v$  with  $f(u) = f(v)$  there is a vertex  $w$  in the path with  $f(w) > f(u)$ . In this definition, we call the value  $f(v)$  the rank of the vertex  $v$ . A vertex ranking of  $G$  is optimal if the largest rank assigned is the smallest in value among all vertex rankings of  $G$ . The vertex ranking number  $r(G)$  is the largest rank used in an optimal vertex ranking. The vertex ranking problem is to determine the vertex ranking number  $r(G)$  of a given graph  $G$ . The edge ranking problem can be defined analogously except that the mapping  $f$  is now from the edge set to the set of all nature numbers.

In the literature, vertex ranking numbers are determined for paths, cycles and cographs. There are also polynomial-time algorithms for the vertex ranking problem and the edge ranking problem on trees. In this thesis, we give a simple proof for the correctness of the algorithm for the vertex ranking on trees. We also propose an algorithm which gives an optimal vertex ranking of a block graph. Finally, we establish results for the vertex ranking problem on cacti.

Keywords: ranking, vertex ranking.

# Study into Fully Orientable Graphs

Hsin-Hao Lai 賴欣豪

Department of Mathematics,

National Taiwan University

f89221010@ntu.edu.tw

ADVISOR: Ko-Wei Lih 李國偉

Assume that  $D$  is an acyclic orientation of a graph  $G$ . An arc is dependent if its reversal creates a directed cycle. Let  $d(D)$  be the number of dependent arcs in  $D$ . Let  $d_{\min}(G)$  ( $d_{\max}(G)$ ) be the minimum (maximum) number of dependent arcs in all acyclic orientations of  $G$ . A graph  $G$  is said to be fully orientable if, for each integer  $d$  satisfying  $d_{\min}(G) \leq d \leq d_{\max}(G)$ , there is an acyclic orientation  $D$  of  $G$  with  $d(D) = d$ .

We begin this thesis by introducing basic definitions, notation, and known results about fully orientability of graphs. In order to characterize  $d_{\min}(G)$ , we then introduce several parameters about triangles and covering graphs. A graph is called a covering graph if it is the underlying graph of the Hasse diagram of a partially ordered set.

We generalize results in Collins and Tysdal [1] about  $d_{\min}(M_m(G))$  of generalized Mycielski graphs  $M_m(G)$ . A method to construct generalized Mycielski graphs  $M_m(G)$  with  $d_{\min}(M_m(G)) = 1$  is also given.

We discuss the following graph operations that preserve fully orientability: the union of two graphs whose intersection is an edge, addition of a path of length at least 2 and addition of an edge between two vertices of degree 2 with a unique common neighbor. We introduce a graph operation called adding a skirt in the following manner. We add a new cycle to a given graph. For each vertex in the cycle, add at most one edge incident to a vertex in the given graph. Except one case, we can prove that the new graph operation preserving fully orientability.

We generalize the color-first tree algorithm in Fisher, Fraughnaugh, Langley and West [2] to obtain the following stronger result. For each spanning tree  $T$  obtained by depth-first search, there exists an integer  $k_T$  such that, for each  $d$  satisfying  $k_T \leq d \leq d_{\max}(G)$ , there is an acyclic orientation  $D$  of  $G$  with  $d(D) = d$ .

A graph is called 2-degenerate if every subgraph has a vertex of degree at most two. A Halin graph is a plane graph obtained by drawing a tree without vertex of degree 2 in the plane, then drawing a cycle through all leaves in the plane. We prove that 2-degenerate graphs, Halin graphs, graphs with

maximum degree at most 3 and graphs with  $d_{\min}(G) \leq 1$  are fully orientable. Furthermore, we characterize  $d_{\min}(G)$  of these graphs.

In the final chapter, we give brief conclusions and pose some open problems for further study.

Keywords: 2-degenerate graphs, acyclic orientations, dependent arcs, fully orientable graphs, Halin graphs..

## References

- [1] K. L. Collins and K. Tysdal, Dependent edges in Mycielski graphs and 4-colorings of 4-skeletons, *J. Graph Theory* 46 (2004), 285–296.
- [2] D. C. Fisher, K. Fraughnaugh, L. Langley and D. B. West, The number of dependent arcs in an acyclic orientation, *J. Combin. Theory Ser. B* 71 (1997), 73–78.

## Transversals in $m \times n$ Arrays

Chang-Chun Lee 李張圳  
Department of Applied Mathematics,  
National Chiao Tung University  
guess166296@yahoo.com.tw

ADVISOR: Hung-Lin Fu 傅恆霖

An  $m$  by  $n$  array consists of  $mn$  cells in  $m$  rows and  $n$  columns, where  $2 \leq m \leq n$ . A partial transversal in an  $m$  by  $n$  array is a set of  $m$  cells, one from each row and no two from the same column. A transversal in an  $m$  by  $n$  array is a partial transversal which  $m$  symbols are distinct. Define  $L(m, n)$  as the largest integer such that if each symbol in an  $m$  by  $n$  array appears at most  $L(m, n)$  times, then the array must have a transversal. In this thesis, we extend the study of finding transversals in a Latin square to find transversals in  $m \times n$  arrays. Mainly, we are interested in determining the value  $L(m, n)$  for certain pairs of positive integers  $m$  and  $n$ .

Keywords: transversal, array.

# A Study of Berge's Strong Path Partition Conjecture

Yen-Ting Chang 張雁婷

Department of Applied Mathematics,

National Chiao Tung University

yanting.chang@gmail.com

ADVISOR: Hung-Lin Fu 傅恆霖

A family  $\mathcal{P} = \{P_1, P_2, \dots, P_m\}$  of paths is called a path partition of  $G$  if its members are vertex disjoint and  $V[\mathcal{P}] = V(G)$ . Let  $k$  be a positive integer, then the  $k$ -norm of a path partition  $\mathcal{P}$  is defined by  $|\mathcal{P}|_k = \sum_{i=1}^m \min\{|P_i|, k\}$ . A path partition  $\mathcal{P}$  minimizes  $|\mathcal{P}|_k$  is called  $k$ -optimal.

A  $k$ -coloring of  $G$  is a family  $\mathcal{C}^k = \{C_1, C_2, \dots, C_k\}$  of  $k$  vertex disjoint independent sets called color classes. A  $k$ -coloring  $\mathcal{C}^k$  is orthogonal to a path partition  $\mathcal{P} = \{P_1, P_2, \dots, P_m\}$  if  $\mathcal{C}^k$  meets every path in  $\mathcal{P}$  in  $\min\{|P_i|, k\}$  different color classes. Berge conjectured that for every  $k$ -optimal path partition  $\mathcal{P}$ , there exists a  $k$ -coloring orthogonal to it, and this is known as Berge's strong path partition conjecture.

This conjecture is still open today, but several results have been obtained in some special cases. In this thesis, we verify this conjecture to be true for certain special digraphs.

Keywords: Berge, path partition.

是否可以考慮 linear  $k$ -chromicity 在 directed graph 上.



# Decomposing the Complete Graph into Paths

Shu-Ren Zhang 張澍仁

Department of Applied Mathematics,

National Chiao Tung University

u890224@alumni.nthu.edu.tw

ADVISOR: Hung-Lin Fu 傅恆霖

It is known that if  $m \mid \binom{v}{2}$ , then the complete graph  $K_v$  can be decomposed into paths of length  $m$  as long as  $1 \leq m \leq v - 1$ . But, for a given positive integer  $1 \leq m \leq v - 1$ ,  $m$  may not be a factor of  $\binom{v}{2}$ . Therefore, we are interested in the case  $m \nmid \binom{v}{2}$ . In these cases, we need a path of distinct length. Let  $P_t$  denote a path with  $t$  edges. Then, it is proved in this thesis that the complete graph  $K_v$  can be decomposed into  $k$   $P_m$ 's and one  $P_r$  if and only if  $\binom{v}{2} = km + r$  where  $0 \leq r < m \leq v - 1$ .

Keywords: complete graph, decomposition, path.

# Diameters of Generalized De Bruijn Graphs

Jyh-Min Kuo 郭志銘

Department of Applied Mathematics,

National Chiao Tung University

jyhminkuo@gmail.com

ADVISOR: Hung-Lin Fu 傅恆霖

The generalized de Bruijn digraph denoted by  $G_B(n, m)$  is the digraph  $(V, A)$  where  $V = \{0, 1, \dots, m-1\}$  and  $(i, j) \in A$  if and only if  $j \equiv in + \alpha \pmod{m}$  for some  $\alpha \in \{0, 1, 2, \dots, n-1\}$ . By replacing each arc of  $G_B(n, m)$  with an undirected edge and eliminating loops and multi-edges, we obtain a generalized undirected de Bruijn graph  $UG_B(n, m)$ . In this article, we prove that for each  $n \geq 2$  and  $n^2 + (\frac{\sqrt{5}+1}{2})n \leq m \leq n^3$  the diameter of  $UG_B(n, m)$  is equal to 3. We also point out there are pairs  $(n, m)$  with  $n^2 < m < n^2 + (\frac{\sqrt{5}+1}{2})n$  such that the diameter of  $UG_B(n, m)$  can be 2 or 3.

Keywords: de Bruijn graph, generalized de Bruijn graph, diameter.

## Cyclic Triples

Yi-Ting Chen 陳宜廷

Department of Applied Mathematics,  
National Chiao Tung University  
u890238@alumni.nthu.edu.tw

ADVISOR: Chih-Wen Weng 翁志文

Let  $\mathbb{C}$  denote the complex field and let  $d$  be a positive integer. We essentially determine all the triples  $A, B, C$  of  $(d+1) \times (d+1)$  matrices over  $\mathbb{C}$  that satisfy

$$A^{d+1} = \alpha I, B^{d+1} = \beta I, C^{d+1} = \gamma I, BA = qAB, CB = qBC, AC = qCA$$

for some nonzero complex numbers  $\alpha, \beta, \gamma$ , and a primitive root  $q$  of unity of order  $d+1$ .

Keywords: cyclic, linear transformation.

# Optimal All-to-All Personalized Exchange in General Shuffle-Exchange Networks

Richard B. Chen 陳柏澍

Department of Applied Mathematics,

National Chiao Tung University

chen.poshu@gmail.com

ADVISOR: Chiuyuan Chen 陳秋媛

All-to-all personalized exchange communication has been widely applied in many parallel and distributed processing applications. By the Latin square method, Yang and Wang proposed an optimal all-to-all personalized exchange algorithm for the unique-path, self-routable multistage interconnection networks (MINs). All the networks considered by Yang and Wang, including the famous shuffle-exchange networks, satisfy  $N = 2^{n+1}$ , in which  $N$  is the number of inputs (outputs) and  $n + 1$  is the number of stages of the network. Do notice that Yang and Wang's algorithm requires the states of all the switches of a stage to be identical; i.e., the stage control technique is used. Padmanabham proposed the general shuffle-exchange network (GSEN) with  $2^n < N \leq 2^{n+1}$ . Since a GSEN is not necessarily a unique-path MIN, Yang and Wang's algorithm may not apply. The purpose of this paper is to propose two optimal all-to-all personalized exchange algorithms for GSENs. Unlike Yang and Wang's algorithm, we abandon the Latin square method and the requirement on the unique-path property. The first algorithm uses the stage control technique and works for arbitrary  $N$ . We will prove it is optimal when the stage control technique is assumed for  $2^{n-1} + 2^n \leq N \leq 2^{n+1}$ . On the contrary, the second algorithm does not use the stage control technique and works only for  $N = 2^n + 2$ . We will prove that it is optimal.

Keywords: multistage interconnection network, parallel and distributed computing, all-to-all communication, all-to-all personalized exchange.

# The Oriented- $k$ -graceful Labelings of Graph

Lian-Jhih Hong 洪連志

Department of Applied Mathematics,

National Dong Hwa University

m9311006@em93.ndhu.edu.tw

ADVISOR: David Kuo 郭大衛

Let  $G$  be a graph with  $|V(G)| = n$  and  $|E(G)| = m$ , a *graceful labeling* of is a one-to-one function  $f : V(G) \rightarrow \{0, 1, 2, \dots, m\}$  such that  $\{|f(u) - f(v)| : uv \in E(G)\} = \{1, 2, \dots, m\}$ . A graph is graceful if it has a graceful labeling. Given integers  $n, k$  with  $k \geq 2$ , we use  $n_k$  to denote the number  $n \bmod k$ , and use  $n_{/k,i}$  to denote the number  $\lfloor \frac{n}{k} \rfloor + \delta_i$ , where

$$\delta_i = \begin{cases} 1, & \text{if } i \leq ((m+1) \bmod k) - 1, \\ 0, & \text{if } i \geq (m+1) \bmod k. \end{cases}$$

For a given graph  $G$ , an *orientation* of  $G$  is a digraph  $D$  obtained from  $G$  by choosing an orientation ( $x \rightarrow y$  or  $y \rightarrow x$ ) for each edge  $xy \in E(G)$ . For a digraph  $D$ , a labeling  $f : V(D) \rightarrow \{0, 1, 2, \dots, k-1\}$  is called a  *$k$ -graceful labeling* of  $D$  if  $\max\{(m+1)_{/k,i} + n - m - 1, 0\} \leq |f^{-1}(i)| \leq (m+1)_{/k,i}$  for each  $i$ ,  $0 \leq i \leq k-1$ , and for all  $j$ ,  $0 \leq j \leq k-1$ , the sets  $A_j = \{uv : (f(v) - f(u))_k = j, uv \in E(D)\}$  satisfy the condition that  $\lfloor \frac{m}{k} \rfloor \leq |A_0| \leq |A_{k-1}| \leq |A_{k-2}| \leq \dots \leq |A_2| \leq |A_1| \leq \lceil \frac{m}{k} \rceil$ . And we say that  $G$  is *oriented- $k$ -graceful* if there exists an acyclic orientation  $D$  of  $G$  which has a  $k$ -graceful labeling  $f$  defined on it.

In this thesis, we fix some notation and terminologies and derive some basic properties in Section 2. And we show that every tree is oriented-2-graceful, oriented-3-graceful and oriented-4-graceful in Section 3. In Section 4, we give necessary and sufficient conditions for union of cycle to be oriented-2-graceful, oriented-3-graceful and oriented-4-graceful.

# The List- $L(2,1)$ -labeling Problem of Digraphs

Jyun-Wei Huang 黃俊瑋

Department of Applied Mathematics,

National Dong Hwa University

m9311003@em93.ndhu.edu.tw

ADVISOR: David Kuo 郭大衛

Given a graph  $G$  with  $n$  vertices and a function  $L : V(G) \rightarrow 2^{\mathbb{N}}$ , let  $A_L = \bigcup_{v \in V(G)} L(v)$ , we say that  $L$  is  $(2,1)$ -choosable for  $G$  if there exists a function  $c : \mathfrak{S} = \{L(v_i) : 1 \leq i \leq n\} \rightarrow A_L$ ,  $c(L(v_i)) = a_i$  for all  $i$ ,  $1 \leq i \leq n$ , which satisfies the following conditions:

- (1)  $a_i \in L(v_i)$ ,
- (2)  $|a_i - a_j| \geq 2$  if  $d_G(v_i, v_j) = 1$ ,
- (3)  $|a_i - a_j| \geq 1$  if  $d_G(v_i, v_j) = 2$ .

In this case, the function  $c$  is said to be a  $(2,1)$ -choosable function of  $G$  with respect to  $L$ . If for all the function  $L$  with  $|L(v_i)| \geq k$  for all  $v_i \in V(G)$ , there is a  $(2,1)$ -choosable function of  $G$  with respect to  $L$ , then we say that  $G$  has a  $k$ -list- $L(2,1)$ -labeling. The list- $L(2,1)$ -labeling number of  $G$ , denoted by  $\lambda_l(G)$ , is defined by  $\lambda_l(G) = \min\{k : G \text{ has a } k\text{-list-}L(2,1)\text{-labeling}\}$ .

We considered the case when the transmitters have direction constraints, that is, the list- $L(2,1)$ -labeling on digraphs. Recall that in a digraph  $D$  the distance  $d_D(x, y)$  from vertex  $x$  to vertex  $y$  is the length of a shortest dipath (directed path) from  $x$  to  $y$ . We then may define list- $L(2,1)$ -labeling,  $k$ -list- $L(2,1)$ -labelings and list- $L(2,1)$ -labeling numbers for digraphs in precisely the same way as for graphs. However, to distinguish with the notation for graphs, we use  $\overrightarrow{\lambda}_l(D)$  for the list- $L(2,1)$ -labeling number of a digraph  $D$ .

In this paper, we study the list- $L(2,1)$ -labeling number of digraphs. We give some basic properties for the list- $L(2,1)$ -labeling number of digraphs in Section two, and consider the list- $L(2,1)$ -labeling number of those digraphs  $D$  whose underline graphs are paths, cycles or trees in Section three. And in the last section, we give the exact value of the list- $L(2,1)$ -labeling number of the digraph  $\overrightarrow{K}_{2,n}$ .

Keywords: list- $L(2,1)$ -labeling, digraph, dipath, path, cycle, tree.

# The $L(3, 2, 1)$ -Labeling of Graphs

Ma-Lian Chia 蔡馬良

Department of Applied Mathematics,

National Dong Hwa University

d9011001@mail.ndhu.edu.tw

ADVISOR: David Kuo 郭大衛

Given a graph  $G$ , the  $L(3, 2, 1)$ -labeling of  $G$  is a function  $f$  from the vertex set  $V(G)$  to the set of all nonnegative integers such that  $|f(u) - f(v)| \geq 1$  if  $d(u, v) = 3$ ,  $|f(u) - f(v)| \geq 2$  if  $d(u, v) = 2$  and  $|f(u) - f(v)| \geq 3$  if  $d(u, v) = 1$ . For a nonnegative integer  $k$ , a  $k$ - $L(3, 2, 1)$ -labeling is an  $L(3, 2, 1)$ -labeling such that no label is greater than  $k$ . The  $L(3, 2, 1)$ -labeling number of  $G$ , denoted by  $\lambda_{3,2,1}(G)$ , is the smallest number  $k$  such that  $G$  has a  $k$ - $L(3, 2, 1)$ -labeling.

In this thesis, we study the  $L(3, 2, 1)$ -labeling numbers of several classes of graphs. We give some basic properties, and give upper bounds for the  $L(3, 2, 1)$ -labeling numbers of general graphs and trees. Following, we study the the  $L(3, 2, 1)$ -labeling numbers of Cartesian product of paths and cycles and the power of paths. Then, we consider the  $L(3, 2, 1)$ -labeling numbers of those graphs which are obtained from the union, join, and Corona of some special graphs. And, in the last section, we determined the  $L(3, 2, 1)$ -labeling numbers of a special class of graphs-the Sierpiński graphs.

Keywords:  $L(3, 2, 1)$ -labeling, Cartesian product, cograph, complete graph, corona, cycle, path, power of paths, Sierpiński graphs, trees.

# Global Defensive Alliance of Trees and Cartesian Product of Graphs

Li-Ling Lai 賴麗鈴

Department of Applied Mathematics,

National Dong Hwa University

m9411008@em94.ndhu.edu.tw

ADVISOR: David Kuo 郭大衛

We study the global defensive alliance of graphs in this thesis. we give an algorithm to determine the global defensive alliance number of trees, and find  $\gamma_a(T)$ , when  $T$  is a complete  $k$ -ary tree for  $k = 2, 3, 4$ . We also study the global defensive alliance of the Cartesian product of paths and cycles. We give upper bounds and lower bounds for  $\gamma_a(P_m \times P_n)$ ,  $\gamma_a(C_m \times P_n)$  and  $\gamma_a(C_m \times C_n)$ , and show that the bounds are sharp for special  $m, n$

Keywords: Global defensive alliance, complete  $k$ -ary tree, Cartesian product.



# The Unique Colorability of a Uniform Mixed Hypergraph

游喬任

Department of Mathematical Sciences,

National Chengchi University

94751003@nccu.edu.tw

ADVISOR: Yi-Wu Chang 張宜武

In this thesis, we find the lower bounds of number of vertices and edges of uniform mixed hypergraph which is uniquely colorable. We show that the size of vertex set of uniform mixed hypergraphs with unique coloring is more than  $(l-1)(m-1)+1$  and we come up a way to construct uniquely colorable uniform mixed hypergraphs. If a mixed hypergraph is an  $r$ -uniform hypergraph with  $\mathcal{D}$  empty, then it is uniquely colorable when  $r = 2$ . Otherwise, an  $r$ -uniform hypergraph with  $\mathcal{D}$  empty is not uniquely colorable. We will introduce two systematic ways to construct a uniform mixed hypergraph which is uniquely colorable and achieves our bounds. First, we reduce the number of  $\mathcal{C}$ -edges such that uniform mixed hypergraphs keep being uniquely colorable. Then we reduce the number of  $\mathcal{D}$ -edges. We consider  $r$ -uniform  $\mathcal{C}$ -hypergraphs and  $\mathcal{D}$ -hypergraphs and find the bounds on their number of edges.

Keywords: hypergraphs, uniquely colorable.

# 完全三分圖分割成星狀圖的探討

李維鴻

Department of Mathematics,  
TamKang University  
leevhome@yahoo.com.tw

ADVISOR: Chin-Mei Kau Fu 高金美

在本篇論文中，我們主要是探討一個完全三分圖 $K_{p,q,r}$ 是否能分割成三角星狀圖(簡稱星狀圖)。

我們先獲得 $K_{p,q,r}$ 可以分割成星狀圖的必要條件，同時利用拉丁方陣進而得到若 $K_{p,q,r}$ 可以分割成星狀圖，則 $K_{np,nq,nr}$ 亦可以分割成星狀圖，對於一些特殊型態的 $p$ 、 $q$ 、 $r$ ，我們獲得 $K_{p,q,r}$ 的星狀圖分割，同時我們將所有 $q$ 為6的倍數且 $q \geq r \geq \frac{q}{2}$ 、 $\frac{5q}{2} \geq p \geq q$ 的 $K_{q,q,r}$ 與 $K_{p,q,q}$ 分割成星狀圖，最後我們給予 $K_{2n,2n,2n}$ 分割成循環星狀圖的建構法。

# Distance Labeling on Cartesian Products of Elementary Graphs

Yem-Hsuan Sung 宋彥萱  
Department of Applied Mathematics,  
Feng Chia University  
m9315439@fcu.edu.tw

ADVISOR: Roger K. Yeh 葉光清

An interesting graph labeling problem comes from the radio channel assignment problem, as well as code assignment in computer networks. One version of the radio channel assignment problem is to assign integer “channels” to a network of transmitters with distance restrictions, such that several levels of interference between nearby transmitters are avoided and the “span” of the labels used is minimized.

Given a graph  $G = (V, E)$  and two numbers  $d_1$  and  $d_2$ , an  $L(d_1, d_2)$ -labeling of  $G$  is an assignment  $f, f: V \rightarrow \{0, 1, \dots\}$  such that  $|f(u) - f(v)| \geq d_i$  if the distance between  $u$  and  $v$  is  $i$  in  $G$ , for  $i = 1, 2$ . The  $L(d_1, d_2)$ -number  $\lambda(G; d_1, d_2)$  (or  $\lambda_{d_1, d_2}(G)$  in some articles.) of  $G$  is the smallest number  $k$  such that there is an  $L(d_1, d_2)$ -labeling of  $G$  using the largest label  $k$ .

A variation of the problem to code assignment in computer networks, i.e. to assignment integer “control codes” to a network of computer stations with distance restrictions, which allow  $d_1 \leq d_2$ .

This thesis studies the  $L(0,1)$ -labeling and the  $L(1,1)$ -labeling on the Cartesian product of elementary graphs.

# A Study on A Magic Labeling Problem

沈志文

Department of Applied Mathematics,

Feng Chia University

yshinbbbb@yahoo.com.tw

ADVISOR: Roger K. Yeh 葉光清

Let  $G = (V, E)$  be a connected simple graph. For any nontrivial abelian group  $A$  (written additively), let  $A^* = A \setminus \{0\}$ . A function  $f: E \rightarrow A^*$  is called a *labeling* of  $G$ . Any such labeling induces a map  $f^+: V \rightarrow A$ , defined by  $f^+(v) = \sum f(uv)$ , where the sum is over all  $uv \in E$ . If there is a labeling  $f$  whose induced map on  $V$  is a constant map, we say that  $f$  is an *A-magic labeling* and that  $G$  is an *A-magic graph*. The corresponding constant is called an *A-magic value*. The *integer-magic spectrum* of a graph  $G$  is the set  $IM(G) = \{k \in \mathbb{N} \mid G \text{ is } Z_k\text{-magic}\}$ , where  $\mathbb{N}$  is the set of all positive integers.

This thesis studies the integer-magic spectrum on several classes of graph.

## Radio 3-Colorings of Lattice Graphs

Cian-Hui Yang 楊千慧

Department of Applied Mathematics,

Feng Chia University

hoby520@hotmail.com

ADVISOR: Roger K. Yeh 葉光清

A major question in the channel (or frequency) assignment problem is how many channels are needed under our constraints. In order to answer this question, some researchers formulate it as a graph coloring (or labeling) problem. It can be stated as follows:

Given integers  $d_1 \geq d_2 \geq d_3 \geq \dots \geq d_r \geq 1$ , for some  $r \geq 2$ , an  $L(d_1, d_2, d_3, \dots, d_r)$ -labeling of a graph  $G = (V, E)$  is a function  $f, f : V \rightarrow \{0, 1, 2, \dots\}$  such that  $|f(u) - f(v)| \geq d_i$  whenever the distance between  $u$  and  $v$  is  $i$  in  $G$ , for  $i = 1, 2, 3, \dots, r$ . The  $L(d_1, d_2, d_3, \dots, d_r)$ -number of  $G$ ,  $\lambda(G; d_1, d_2, d_3, \dots, d_r)$ , is the smallest number  $k$  such that there is an  $L(d_1, d_2, d_3, \dots, d_r)$ -labeling on  $G$  using the largest label  $k$ .

An  $L(d, d - 1, d - 2, \dots, 2, 1)$ -labeling ( $d \geq 2$ ) is also called a *radio  $d$ -coloring* in some literatures. This thesis will study the radio 3-coloring in some classes of graphs.

Keywords: graph coloring, radio, labeling.

# Graph Labelings with Three Levels of Constraints

Honya Liao 廖虹雅

Department of Applied Mathematics,

Feng Chia University

honya1201@yahoo.com.tw

ADVISOR: Roger K. Yeh 葉光清

Motivated by the channel assignment problem, the following graph labeling has been proposed: Given integers  $d_1 \geq d_2 \geq d_3 \geq \cdots \geq d_r \geq 1$ , for some  $r \geq 2$ , an  $L(d_1, d_2, d_3, \dots, d_r)$ -labeling of a graph  $G = (V, E)$  is a function  $f, f : V \rightarrow \{0, 1, 2, \dots\}$  such that  $|f(u) - f(v)| \geq d_i$  whenever the distance between  $u$  and  $v$  is  $i$  in  $G$ , for  $i = 1, 2, 3, \dots, r$ . The  $L(d_1, d_2, d_3, \dots, d_r)$ -number of  $G$ ,  $\lambda(G; d_1, d_2, d_3, \dots, d_r)$ , is the smallest number  $k$  such that there is an  $L(d_1, d_2, d_3, \dots, d_r)$ -labeling on  $G$  using the largest label  $k$ . This article studies the labelings with constraints  $(d, 1, 1)$  and  $(3, 2, 1)$  on several classes of graphs.

Keywords:  $L(2, 1)$ -labeling.

# Finding Hamiltonian Cycle on Perfect Matching of Hypotraceable Graphs

Jin-Long Lin 林金龍

Department of Computer Science and Information Engineering,  
National Chiayi University  
s0950319@mail.ncyu.edu.tw

ADVISOR: Yung-Ling Lai 賴泳伶

The interconnected network is usually represented by a graph  $G$  where the vertices represent processors and the edges represent links between processors. A hypotraceable graph  $G$  is a graph with no Hamiltonian path, and for any vertex  $v \in V(G)$ ,  $G - v$  has a Hamiltonian path. This paper investigates a perfect matching of two or more hypotraceable graphs. It has been shown that a perfect matching of two hypotraceable graphs and a serial matching of  $n$  hypotraceable graphs with the same order are all having the Hamiltonian property.

Keywords: Hamiltonian, hypotraceable, perfect matching.

# A Study on Antipodal Labellings Problems

Jyun-Nan Ou 歐俊男

Department of Computer Science and Information Engineering,  
National Chi Nan University  
ten.you@msa.hinet.net

ADVISOR: Justie Su-Tzu Juan 阮夙姿

An antipodal labelling of  $G$  with diameter  $d$  is a function  $f$  that assigns a non-negative integer to the vertices of  $G$  such that for any vertices  $u$  and  $v$  satisfying  $|f(u) - f(v)| \geq d - d(u, v)$ , where  $d(u, v)$  is the distance between  $u$  and  $v$ . The span of an antipodal labelling  $f$  is  $\max\{f(u) - f(v) : u, v \in V(G)\}$ . The antipodal number of  $G$ , denoted by  $\text{an}(G)$ , is the minimum span of all antipodal labellings for  $G$ . Let  $T$  be a tree and  $S_{l_1, l_2, l_3, \dots, l_n}$  with  $l_1 \geq l_2 \geq \dots \geq l_n, n \geq 3$  be a spider (tree with at most one vertex of degree more than two). In the thesis, we give two lower bounds for tree and spider, respectively. We achieve the bound on some special spider. Besides, we also give two lower bounds and two upper bounds for  $P_2 \times P_n$  and  $P_2 \times C_n$ , respectively.

Keywords: Channel assignment problem, radio  $k$ -labelling, antipodal labelling, radio number, antipodal number.



## 2007組合數學(新苗)研討會與會人員名單

(依服務單位、身份、姓名排列)

姓名	身份	服務單位	E-mail
Louis Esperet	學生	LaBRI - Université Bordeaux I	esperet@labri.fr
Nicolas Roussel	學生	LaBRI - Université Bordeaux I 中山大學應用數學系	nrous@hotmail.com
江南波	老師	大同大學應用數學系	npchian@ttu.edu.tw
張薰文	老師	大同大學應用數學系	hwchang@ttu.edu.tw
林耀仁	學生	大同大學應用數學系	jacky5500kimo@yahoo.com.tw
陳俊達	學生	大同大學應用數學系	u9008015@mail.ttu.edu.tw
曾湘寧	學生	大同大學應用數學系	et046059@yahoo.com.tw
官大智	老師	中山大學資訊工程學系	guan@cse.nsysu.edu.tw
陳秉宏	學生	中山大學資訊工程學系	ud17g@yahoo.com.tw
王彩蓮	老師	中山大學應用數學系	tlwong@math.nsysu.edu.tw
朱緒鼎	老師	中山大學應用數學系	zhu@math.nsysu.edu.tw
董立大	老師	中山大學應用數學系	ldtong@math.nsysu.edu.tw
王鴻志	學生	中山大學應用數學系	p922040002@student.nsysu.edu.tw
林銘宏	學生	中山大學應用數學系	m942040010@student.nsysu.edu.tw
紀文德	學生	中山大學應用數學系	m952040020@student.nsysu.edu.tw
張宏鏞	學生	中山大學應用數學系	gianni.chang@msa.hinet.net
張定邦	學生	中山大學應用數學系	dibonchang@yahoo.com.tw
張家榮	學生	中山大學應用數學系	m952040009@student.nsysu.edu.tw
楊宗穎	學生	中山大學應用數學系	yangcy@math.nsysu.edu.tw
顏珮嵐	學生	中山大學應用數學系	yenpl@math.nsysu.edu.tw

## 2007組合數學(新苗)研討會與會人員名單

姓名	身份	服務單位	E-mail
蘇芳媚	學生	中山大學應用數學系	andysue8787@hotmail.com
陳又瑄		中山大學應用數學系	chenys@math.nsysu.edu.tw
劉尙存		中山大學應用數學系	liust@math.nsysu.edu.tw
林 強	老師	中央大學數學系	lchiang@math.ncu.edu.tw
葉鴻國	老師	中央大學數學系	hgyeh@math.ncu.edu.tw
廖勝強	老師	中央大學數學系	scliaw@math.ncu.edu.tw
張凱涵	學生	中央大學數學系	942201005@cc.ncu.edu.tw
張凱媛	學生	中央大學數學系	mypenny52@yahoo.com.tw
陳億庭	學生	中央大學數學系	u8013800@cc.ncu.edu.tw
葉政峰	學生	中央大學數學系	942201013@cc.ncu.edu.tw
羅勝鴻	學生	中央大學數學系	942201013@cc.ncu.edu.tw
李國偉	老師	中央研究院數學所	makwlih@sinica.edu.tw
陳世晏	學生	中原大學應用數學系	yan@blsh.tp.edu.tw
曹友賓	老師	中國科技大學	u8822806@math.nctu.edu.tw
陳哲焜	老師	中國科技大學	jungle@cute.edu.tw
李宣助	老師	仁德醫專	k0401001@ms4.kntech.com.tw
黃明輝	老師	元培科技大學	rudin86@yahoo.com.tw
陳伯亮	老師	台中技術學院	blchen@ntit.edu.tw
張鎮華	老師	台灣大學數學系	gjchang@math.ntu.edu.tw
吳佼佼	博士後	台灣大學數學系	wujj0007@yahoo.com.tw
趙永強	博士後	台灣大學數學系	yqzhao@sina.com
吳欣融	學生	台灣大學數學系	r93221031@ntu.edu.tw
林武雄	學生	台灣大學數學系	r901001@math.ntu.edu.tw

## 2007組合數學(新苗)研討會與會人員名單

姓名	身份	服務單位	E-mail
黃良豪	學生	台灣大學數學系	r90221003@ntu.edu.tw
鄧乃心	學生	台灣大學數學系	r94221021@ntu.edu.tw
賴欣豪	學生	台灣大學數學系	f89221010@ntu.edu.tw
徐泰煒	老師	台灣師範大學數理學科	twhsu@ntnu.edu.tw
洪蓉婷	老師	左營國中	m922040012@alumni.nsysu.edu.tw
郭君逸	老師	交通大學	davidguo.am90g@nctu.edu.tw
林威雄	學生	交通大學	gauss1980@hotmail.com
邱鈺傑	學生	交通大學	well.am94g@nctu.edu.tw
石園鋼	學生	交通大學資訊科學暨工程研究所	ykshih@cs.nctu.edu.tw
翁志文	老師	交通大學應用數學系	weng@math.nctu.edu.tw
陳秋媛	老師	交通大學應用數學系	cychen@mail.nctu.edu.tw
傅恆霖	老師	交通大學應用數學系	hlfu@math.nctu.edu.tw
卜文強	學生	交通大學應用數學系	freedomile@yahoo.com.tw
吳政軒	學生	交通大學應用數學系	wuzsh.am95g@nctu.edu.tw
李張圳	學生	交通大學應用數學系	guess166296@yahoo.com.tw
張雁婷	學生	交通大學應用數學系	yanting.chang@gmail.com
張澍仁	學生	交通大學應用數學系	u890224@alumni.nthu.edu.tw
連敏筠	學生	交通大學應用數學系	lienmy.am94g@nctu.edu.tw
郭志銘	學生	交通大學應用數學系	jyhminkuo@gmail.com
陳子鴻	學生	交通大學應用數學系	x88cth@yahoo.com.tw
陳宏賓	學生	交通大學應用數學系	andan.am92g@nctu.edu.tw
陳宜廷	學生	交通大學應用數學系	u890238@alumni.nthu.edu.tw
陳柏澍	學生	交通大學應用數學系	chen.poshu@gmail.com

## 2007組合數學(新苗)研討會與會人員名單

姓名	身份	服務單位	E-mail
黃志文	學生	交通大學應用數學系	jwh.am91@nctu.edu.tw
黃信菖	學生	交通大學應用數學系	pollow.am95g@nctu.edu.tw
蔡佩純	學生	交通大學應用數學系	mathpure1130@yahoo.com.tw
謝奇璵	學生	交通大學應用數學系	chuo.am95g@nctu.edu.tw
林婉茹	學生	成功大學	b9024031@alumni.nsysu.edu.tw
范慧蘭	老師	竹光國中	huilanfan@yahoo.com.tw
黃文中	老師	東吳大學數學系	wchuang@math.scu.edu.tw
陳弘倫	學生	東吳大學數學系	a0911662239@yahoo.com.tw
劉秋陽	學生	東吳大學數學系	lainain@yahoo.com.tw
鄭俊憲	學生	東吳大學數學系	issacmememe@hotmail.com
張家郎	學生	東華大學	d9411001@em94.ndhu.edu.tw
張翠萍	學生	東華大學	m9411010@em94.ndhu.edu.tw
郭大衛	老師	東華大學應用數學系	davidk@server.am.ndhu.edu.tw
洪連志	學生	東華大學應用數學系	m9311006@em93.ndhu.edu.tw
黃俊瑋	學生	東華大學應用數學系	m9311003@em93.ndhu.edu.tw
蔡幸儒	學生	東華大學應用數學系	m9211009@em92.ndhu.edu.tw
蔡馬良	學生	東華大學應用數學系	d9011001@mail.ndhu.edu.tw
賴麗鈴	學生	東華大學應用數學系	m9411008@em94.ndhu.edu.tw
游喬任	學生	政治大學應用數學系	94751003@nccu.edu.tw
鄭立國	老師	海軍官校資訊管理系	lgjeng@mail.cna.edu.tw
顏經和	老師	真理大學數學系	jhyan@email.au.edu.tw
游森棚	老師	高雄大學應用數學系	senpengeu@gmail.com
張式奇	學生	高雄大學應用數學系	jctyrant@jctyrant.com

## 2007組合數學(新苗)研討會與會人員名單

姓名	身份	服務單位	E-mail
蔡杰政	學生	高雄師範大學數學系	asdf-813@yahoo.com.tw
唐惠欽	老師	高雄應用科技大學工業工程與管理系	tang@cc.kuas.edu.tw
林遠隆	學生	淡江大學	gauss_g@yahoo.com.tw
高金美	老師	淡江大學數學系	cmfu@mail.tku.edu.tw
李維鴻	學生	淡江大學數學系	leevhome@yahoo.com.tw
徐育鋒	學生	淡江大學數學系	joliwugy@yahoo.com.tw
陳昆楠	學生	淡江大學數學系	integer38@yahoo.com.tw
黃彥菱	學生	清華大學資訊工程系	ylhuang@vc.cs.nthu.edu.tw
葉光清	老師	逢甲大學應用數學系	rkyeh@math.fcu.edu.tw
宋彥萱	學生	逢甲大學應用數學系	micky83y@ms27.hinet.net
沈志文	學生	逢甲大學應用數學系	yshinbbbb@yahoo.com.tw
楊千慧	學生	逢甲大學應用數學系	hoby520@hotmail.com
廖虹雅	學生	逢甲大學應用數學系	honya1201@yahoo.com.tw
孫召明	老師	陸軍官校管理科學系	sunmoonobviously@yahoo.com.tw
孫一凡	老師	義守大學應用數學系	ifan@isu.edu.tw
吳信儒	學生	嘉義大學	shinju.wu@msa.hinet.net
張添璋	學生	嘉義大學	powermanteddy@gmail.com
林金龍	學生	嘉義大學資訊工程系	s0950319@mail.ncyu.edu.tw
賴泳伶	老師	嘉義大學資訊工程學系	yllai@mail.ncyu.edu.tw
洪世嘉	學生	嘉義大學資訊工程學系	sallenray@yahoo.com.tw
陳一鳴	學生	嘉義大學資訊工程學系	s0950299@mail.ncyu.edu.tw
張飛黃	老師	嘉義大學應用數學系	feihuang0228@gmail.com
嚴志弘	老師	嘉義大學應用數學系	chyen@mail.ncyu.edu.tw

## 2007組合數學(新苗)研討會與會人員名單

姓名	身份	服務單位	E-mail
林和傑	學生	嘉義大學應用數學系	taiwanjack@gmail.com
陳怡婷	學生	嘉義大學應用數學系	s0921925@mail.ncyu.edu.tw
王怡君	學生	暨南大學資訊工程學系	s94321520@ncnu.edu.tw
吳泰融	學生	暨南大學資訊工程學系	s3321015@ncnu.edu.tw
李青芬	學生	暨南大學資訊工程學系	ching-100@yahoo.com.tw
林佑霖	學生	暨南大學資訊工程學系	maik060@msn.com
劉旭玲	學生	暨南大學資訊工程學系	shyuling@gmail.com
歐俊男	學生	暨南大學資訊工程學系	ten.you@msa.hinet.net
蔡惠嬋	學生	暨南大學資訊工程學系	s95321511@ncnu.edu.tw
阮夙姿	老師	暨南國際大學資訊工程系	jsjuan@ncnu.edu.tw
潘俊杰	老師	輔仁大學數學系	jackpan@math.fju.edu.tw
施孟萱	學生	輔仁大學數學系	soyinkaya@yahoo.com.tw
翁立維	學生	輔仁大學數學系	lyway118@yahoo.com.tw
林承穎	老師	樹德科技大學資訊工程系	lincy@mail.stu.edu.tw
黃國卿	老師	靜宜大學應用數學系	kchuang@pu.edu.tw

\*8/6前所登錄名單

