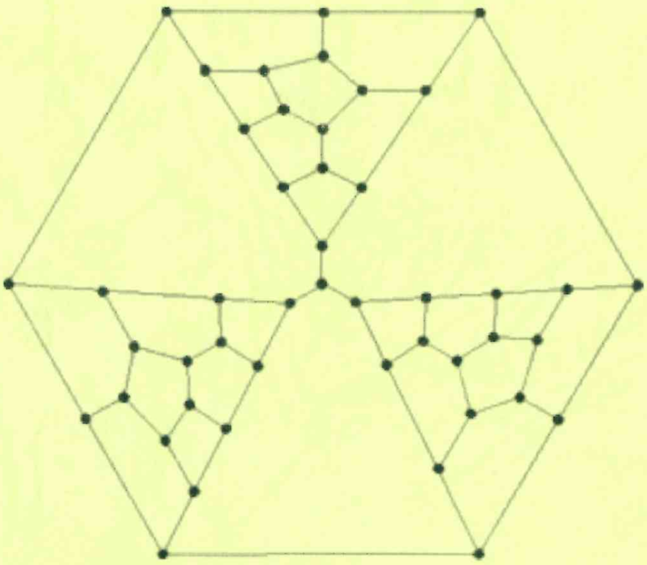
第 18 屆

# 2009 組合數學暨新苗研討會



August 10-11, 2009

主辦單位：中原大學應用數學系

協辦單位：國科會數學研究推動中心

# 第十八屆組合數學暨新苗研討會

**2009 年 8 月 10 日 至 8 月 11 日**

中原大學
應用數學系

主辦單位：中原大學應用數學系

協辦單位：國科會數學研究推動中心

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# 序 言

新苗組合數學研討會迄今已舉辦十七屆（在下一頁的一覽表中我們嘗試重建歷史，缺漏之處還請各位先進提供資料補遺），今年為第十八屆，由中原大學應用數學系主辦。本研討會由李國偉教授（中央研究院數學研究所）發起，並由各大學輪流主辦，是國內組合數學界重要的研討會，讓每年相關領域的碩博士應屆畢業同學有發表和交流的園地，也提供與會者彼此聯絡情誼及相互觀摩的機會。

本研討會共有三場專題演講及二十五場次的口頭論文發表。專題演講部份，我們邀請到交通大學應用數學系黃大原教授、靜宜大學資訊工程系徐力行教授及中央大學數學系林強教授發表演講。感謝三位教授的參與，使本次研討會增色許多。

由衷感謝各界對本次研討會之熱烈參與，其中擔任上一屆主辦學校的交通大學傅恆霖教授及陳秋媛教授不吝分享籌辦經驗、相關檔案及網站，讓我們整體工作輕省不少；感謝靜宜大學應用數學系黃國卿教授及各審稿委員的用心，出國期間仍舊撥冗開會審查，評審出值得肯定的優良論文。此外特別感謝行政院國家科學委員會數學研究推動中心、中原大學研發處、及各贊助廠商（歐亞圖書公司、高立圖書公司、鼎隆圖書股份有限公司）對本研討會的經費補助。期待各位能有愉快的研討會，願上帝賜福各位

平安喜樂！萬事如意！

籌備委員會　高欣欣、李是男、史青林 敬上

2009 年 8 月 10 日

# 台灣組合數學暨新苗研討會歷屆主辦單位

| 屆 別 | 年 | 主 辦 單 位 |
|---|---|---|
| 1 | 1992 | |
| 2 | 1993 | |
| 3 | 1994 | |
| 4 | 1995 | 交通大學 應用數學系 |
| 5 | 1996 | |
| 6 | 1997 | |
| 7 | 1998 | 中山大學 應用數學系 |
| 8 | 1999 | 中央研究院 數學所 |
| 9 | 2000 | 逢甲大學 應用數學系 |
| 10 | 2001 | 淡江大學 數學系 |
| 11 | 2002 | 中央大學 數學系 |
| 12 | 2003 | 台灣大學 數學系 |
| 13 | 2004 | 靜宜大學應用 數學系 |
| 14 | 2005 | 東華大學 應用數學系 |
| 15 | 2006 | 中國科技大學 |
| 16 | 2007 | 中山大學 應用數學系 |
| 17 | 2008 | 交通大學 應用數學系 |
| **18** | **2009** | **中原大學 應用數學系** |

# 2009 組合數學暨新苗研討會 議程

會場：中原大學 全人教育村(北棟四樓) 國際會議廳

**2009 年 8 月 10 日【星期一】**

| | | | |
|---|---|---|---|
| 08:30-09:20 | 辦理報到手續 | | |
| 09:20-09:30 | 開 幕 | | |
| **Session 1** | **主持人：** | **史青林 教授 [中原大學]** | |
| 09:30-10:20 | 大會專題演講 (一) 演講者：黃大原教授 | | |
| 10:20-10:40 | ❀ Tea Break ❀ | | |
| **Session 2** | **主持人：** | **賴泳伶 教授 [嘉義大學]** | |
| 10:40-11:00 | 演講者： | 林武雄 [台灣大學 數學所]<br>Equitable Coloring of Graph Products | **P. 1** |
| 11:00-11:20 | 演講者： | 陳怡婷 [嘉義大學 應用數學所]<br>Equitable Coloring of *k*-regular and 3-partite Graphs | **P. 2** |
| 11:20-11:30 | 休息 | | |
| **Session 3** | **主持人：** | **顏經和 教授[真理大學]** | |
| 11:30-11:50 | 演講者： | 李愛蓮 [中原大學 應用數學所]<br>On $\alpha-$labelings of Odd-degree $(r,2r-4)-$Crowns | **P. 3** |
| 11:50-12:10 | 演講者： | 楊枝滎 [中原大學 應用數學所]<br>On $\alpha-$labelings of Even-degree $(r,2r-4)-$Crowns | **P. 4** |
| 12:10-13:10 | 午餐 | | |
| **Session 4** | **主持人：** | **高欣欣 教授 [中原大學]** | |
| 13:10-14:00 | 大會專題演講 (二) 演講者：徐力行教授 | | |
| 14:00-14:10 | 休息 | | |
| **Session 5** | **主持人：** | **董立大 教授 [中山大學]** | |
| 14:10-14:30 | 演講者： | 劉旭玲 [暨南大學 資訊工程所]<br>A Study on List Coloring Problems | **P. 5** |
| 14:30-14:50 | 演講者： | 楊琇如 [東華大學 應用數學所]<br>$(p,1)-$total Labeling of Complete Graphs | **P. 6** |
| 14:50-15:10 | 演講者： | 洪世嘉 [嘉義大學 資訊工程所]<br>$L(2,1)$ Labeling for $n-$Dimensional Graphs | **P. 7** |
| 15:10-15:30 | ❀ Tea Break ❀ | | |
| **Session 6** | **主持人：** | **郭大衛 教授 [東華大學]** | |
| 15:30-15:50 | 演講者： | 黃元貞 [中央大學 數學所]<br>Some Results on Distance-two Labeling of a Graph | **P. 8** |
| 15:50-16:10 | 演講者： | 劉士慶 [交通大學 應用數學所]<br>On the Study of the Diameter of Chordal Ring Networks | **P. 9** |
| 16:10-16:30 | 演講者： | 曾慧棻 [交通大學 應用數學所]<br>On the All-to-All Personalized Exchange Problem in Chordal Ring Networks | **P. 10** |
| 16:30-16:40 | 休息 | | |
| **Session 7** | **主持人：** | **黃明輝 教授 [元培]** | |
| 16:40-17:00 | 演講者： | 吳宗翰 [大同大學 應用數學所]<br>On Antimagic Labeling of Graphs | **P. 12** |
| 17:00-17:20 | 演講者： | 李青芬 [暨南大學 資訊工程所]<br>A Study of Secret Image Sharing Schemes | **P. 13** |
| 17:20-17:30 | 休息 | | |

| | | | |
|---|---|---|---|
| 17:30-17:50 | 演講者： | 李光祥 [交通大學　應用數學所] An Extension of Stein-Lovász Theorem and Some of Its Applications | P. 15 |
| 17:50-18:10 | 演講者： | 裴若宇 [交通大學　應用數學所] Forbidding Multicolored Cycles in an Edge-colored $K_{m,n}$ | P. 16 |
| 18:30~ | 晚 宴 | | |

**2009 年 8 月 11 日【星期二】**

| | | | |
|---|---|---|---|
| **Session 8** | **主持人：** | **李是男 教授 [中原大學]** | |
| 09:20-10:10 | 大會專題演講 (三) | 演講者：林 強 教授 | |
| 10:10-10:20 | ❀ Tea Break ❀ | | |
| **Session 9** | **主持人：** | **潘俊杰 教授 [輔仁大學]** | |
| 10:20-10:40 | 演講者： | 陳巧玲 [交通大學　應用數學所] The Multiplicity of Laplacian Eigenvalue One | P. 17 |
| 10:40-11:00 | 演講者： | 吳亞倫 [嘉義大學　應用數學所] Characterizing the 2d-nary Banyan-type Networks | P. 18 |
| 11:00-11:20 | 演講者： | 陳一鳴 [嘉義大學　資訊工程所] On Vertex Ranking and Related Problem of Graphs | P. 19 |
| 11:20-11:30 | 休息 | | |
| **Session 10** | **主持人：** | **蔡明春 教授 [中華大學]** | |
| 11:30-11:50 | 演講者： | 陳聖華 [台灣大學　數學所] S-Packing Coloring on Graphs | P. 20 |
| 11:50-12:10 | 演講者： | 游舜婷 [交通大學　應用數學所] A Study of DNA Graphs | P. 21 |
| 12:10-13:10 | 午餐 | | |
| **Session 11** | **主持人：** | **翁志文 教授 [交通大學]** | |
| 13:10-13:30 | 演講者： | Ariane Garon [Université du Québec à Montréal] The Fibonacci Sequence in the Fibonacci Word | P. 22 |
| 13:30-13:50 | 演講者： | 高國華 [暨南大學　資訊工程所] A Quadratic Algorithm for Finding Next-to-shortest Paths in Graphs | P. 23 |
| 13:50-14:10 | 演講者： | 蔡松育 [交通大學　應用數學所] The Rigidity Property and the Unique Localization Problem of Sensor Networks | P. 24 |
| 14:10-14:20 | 休息 | | |
| **Session 12** | **主持人：** | **陳秋媛 教授 [交通大學]** | |
| 14:20-14:40 | 演講者： | 劉宜君 [交通大學　應用數學所] Constructing Independent Spanning Trees for Hypercubes and Locally Twisted Cubes | P. 25 |
| 14:40-15:00 | 演講者： | 王必祥 [中原大學　應用數學所] Mutually Independent Hamiltonian Cycles in $k$-ary $n$-cubes when $k$ is odd | P. 26 |
| 15:00-15:20 | 演講者： | 蔡佳融 [中原大學　應用數學所] The Panpositionable Hamiltonicity of $k$-ary $n$-cubes | P. 27 |
| **15:20-15:40** | **頒獎及交接** | | |

# Pooling Spaces, Four-weight Spin Models and Association Schemes

黃大原* 翁志文（交通大學）
王愷順（北京師大）

2009 組合數學暨新苗研討會
中原大學
2009 年 8 月 10 日

## Abstract

It is interesting to point out the comment made by D.-Z Du（堵丁柱）et.al. in "A Survey on Combinatorial Group Testing Algorithms, DIMACS Proceedings of Medical Applications of Discrete Mathematics 2000" that "We would like to conclude that this (combinatorial group testing algorithms) is a young and interesting field with deep connections to coding theory and design theory. We strongly believe that the theory of *distance regular graphs*（距離正則圖）, particular *association schemes*（結合方案）, should play an important role in improving our pooling designs".

Motivated by the above comments, we will first introduce the notions of distance regular graphs and association schemes, followed by the notions of *pooling spaces* for error-tolerant group testing purpose, and *four-weight spin models* for statistical mechanics. We will then show how the notion of distance regular graphs and association schemes can be used to deal with these two types of problems. A series of our joint works on these topics over the past few years will be surveyed in this talk.

# Hamiltonian Aspects of Interconnection Networks

徐力行 (靜宜大學)

2009 組合數學暨新苗研討會
中原大學
2009 年 8 月 10 日

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## Abstract

In this talk, I will present a survey on hamiltonian properties on iterconnection nections, expecially on fault tolerant hamiltonianicity, fault tolerant pancyclic, spanning connectivity and multually independent hamiltonicity.  Many open problems will be presented.

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# Status Unique Graphs and Degree Unique Graphs

林 強 ( 中央大學)

2009 組合數學暨新苗研討會
中原大學
2009 年 8 月 10 日

## Abstract

## 1. Status Unique Graphs

Let G be a connected graph. The status of any vertex x in G is

$$s(x) = \sum_{y \in V(G)} d(x,y).$$

The status sequence of G is the list of the statuses of all the vertices of G arranged in nondecreasing order.

Let F be a family of connected graphs and G∈F. If G is uniquely determined in F by its status sequence, then we say that G is status unique in F.

**Theorem** *Every spider is status unique in the family of all trees.*

**Conjecture** A tree and a non-tree graph can not have the same status sequence.

## 2. Degree unique graphs

The degree sequence of a graph G is the list of degrees of all the vertices of G arranged in nondecreasing order.

**Example** $P_7$ and $P_4 \cup C_3$ have the same status sequence: 1, 1, 2, 2, 2, 2, 2.

Let F be a family of graphs and G∈F. If G is uniquely determined in F by its degree sequence, then we say that G is degree unique in F.

We have characterizations for graphs which are degree unique in some specific families, for examples, trees, forests, and unicyclic graphs.

# Equitable Colorings of Graph Products

**林武雄** ( Wu-Hsiung Lin )

Department of Mathematics
College of Science
National Taiwan University
Doctoral Dissertation

**ADVISOR: 張鎮華 教授**

## Abstract

For a positive integer $k$, if we can color all vertices of G in $k$ colors such that the colors of two adjacent vertices are different, and any two color classes are different in size at most one, then we call the graph G equitably k-colorable. The equitable chromatic number of a graph G, denoted by $\chi = (G)$, is the minimum $k$ such that G is equitably $k$-colorable. The equitable chromatic threshold of a graph G, denoted by $\chi^* = (G)$ , is the minimum t such that G is equitably k-colorable for $k \geq t$.

Graph products have been widely studied in many actions and parameters of the structure of graphs, including graph colorings. And the equitable coloring is one of the variations of colorings with more applications in real—the frequency of the appearance of all colors are near to each other.

In this thesis we study equitable chromatic numbers and equitable chromatic thresholds of Cartesian products, Kronecker products and strong products of graphs. In particular, we give exact values and upper bounds on equitable chromatic numbers and equitable chromatic thresholds of these three kinds of product of some classes of graphs such as paths, cycles, complete graphs and complete bipartite graphs.

## 中文摘要

對正整數$k$，如果我們能將一個圖$G$中所有的點著$k$種顏色，使得兩個相鄰的點所著的顏色相異，而且任兩種顏色類在數量上差距最多一個，則我們稱圖$G$是均勻$k$-可著的。一個圖$G$的均勻著色數，記為$\chi = (G)$，是最小的$k$使得$G$是均勻$k$-可著的。一個圖$G$的均勻著色臨界值，記為$\chi^* = (G)$，是最小的$t$使得對任何$k$大於$t$，$G$都是均勻$k$-可著的。

乘積圖已在圖上的作用與結構的參數上有廣泛的研究，包括圖著色。而均勻著色是著色的變化型中在現實上有著較多應用之一 — 所有顏色出現的頻率相互接近。

本文中我們研究 Cartesian 乘積圖、Kronecker 乘積圖、與強乘積圖的均勻著色數與均勻著色臨界值。特別地，我們給出路徑、圓圈、完全圖、與完全二部圖的這三類乘積圖均勻著色數與均勻著色臨界值的精確數值與上界。

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# Equitable Coloring of k-regular and 3-partite Graphs

陳怡婷 ( Yi-Ting Chen )

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**ADVISOR: Chih-Hung Yen Ph. D**

## Abstract

A proper $k$-coloring of a graph $G$ is a labeling $f: V(G) \to \{1, 2, \ldots, k\}$ such that adjacent vertices have different labels. The labels are *colors*; the vertices of one color form a *color class*. A graph $G$ is *k-colorable* if there exits a proper $k$-coloring of $G$. The *chromatic number* of a graph $G$, written $\chi(G)$, is the least $k$ such that $G$ is $k$-colorable. A graph $G$ is *k-chromatic* if $\chi(G) = k$. An *equitable k-coloring* of a graph $G$ is a proper $k$-coloring of $G$ such that the sizes of any two color classes differ by at most one. A graph $G$ is *equitably k-colorable* if there exists an equitable $k$-coloring of $G$. Furthermore, a graph $G$ is *k-regular* if all vertices of $G$ have the same degree $k$.

Let $\Delta(G)$ denote the *maximum degree* of a graph $G$. An earlier result of Hajnal and Szemerédi [12] implied that a graph $G$ can be equitably colored with $k$ colors if $k \geq \Delta(G)+1$. Hence, we would like to know whether any given graph $G$ can also be equitable colored by using $\Delta(G)$ colors, namely, be equitably $\Delta(G)$-colorable. In 1997, Yen [23] gave a necessary condition for a graph $G$ with $\Delta(G) \geq \chi(G)$ to be equitably $\Delta(G)$-colorable. Yen also proved that such a condition is sufficient when $G$ is bipartite (or $\chi(G) \leq 2$ ) or $G$ is 4-regular and 3-chromatic.

Hence, in this thesis, we study the necessary and sufficient condition for a $k$-regular and 3-chromatic graph $G$ with $k \geq 3$ to be equitably $k$-colorable. First, we prove that a graph $G$ with $\Delta(G) \geq \max\{\chi(G), |V(G)|/2\}$ is equitably $\Delta(G)$-colorable if and only if $G$ is different from $K_{2n+1,2n+1}$ for any $n \geq 1$. Then we give the necessary and sufficient condition for a graph $G$ with $\Delta(G) = 3 \geq \chi(G)$ to be equitably $\Delta(G)$-colorable. Finally, we have some results on the equitable $\Delta(G)$-coloring for a $k$-regular and 3-chromatic graph $G$ with $k \geq 5$.

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# On α-labelings of Odd-degree (γ,2γ−4)-Crowns


李愛蓮

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**ADVISOR: 史青林 教授**



## Abstract

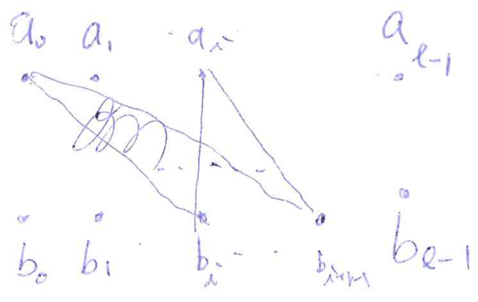
Let $G$ be a graph with $m$ vertices and $n$ edges. A graceful labeling of $G$ is an injection $f : V(G) \to \{0,1,2,\ldots,n\}$ such that when each edge $uv$ is assigned the label $|f(u)-f(v)|$, the resulting edge labels are distinct. We call $f$ is a graceful labeling of $G$.

A graceful labeling is called an $\alpha$-labeling if there exists an integer $\lambda$ such that for each edge $uv$ either $f(u) \le \lambda < f(v)$ or $f(v) \le \lambda < f(u)$. The integer $\lambda$ is called critical value.

Let $r$ and $l$ be two positive integers. A bipartite graph $G$ defined on $A \cup B$, where $A \cap B = \phi$, $A = \{a_0, a_1, a_2, \ldots, a_{l-1}\}$ and $B = \{b_0, b_1, b_2, \ldots, b_{l-1}\}$ is called an $(r,l)$-crown, for each $i \in Z_l$, $a_i$ is adjacent to $b_j$ if and only if

$j \in \{i, i+1, i+2, \ldots, i+r-1\}$ (mod $l$)

In this article, we prove that for each odd integer $r \ge 5$, an odd-degree $(r, 2r-4)$-crown has an $\alpha$-labeling.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# On α-labelings of Even-degree (γ,2γ−4)-Crowns

**楊枝熒**

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**ADVISOR: 史青林 教授**

## Abstract

Let $G$ be a graph with $q$ edges, and $f$ be an injection from $V(G)$ into $\{0,1,2,\ldots,q\}$. We call $f$ is a $\beta$-labeling of $G$ if the values $|f(u)-f(v)|$ for the $q$ pairs of adjacent vertices $u$ and $v$ are distinct. A $\beta$-labeling is now more commonly called a graceful labeling.

An $\alpha$-labeling is a graceful labeling having the additional property that there is an integer $\lambda$ which is called a critical value so that for each edge $\{u,v\}\in E(G)$ either $f(u)\le\lambda<f(v)$ or $f(v)\le\lambda<f(u)$.

A bipartite graph $G$ is defined on $A\cup B$, which $A=\{a_0,a_1,a_2,\ldots,a_{l-1}\}$, $B=\{b_0,b_1,b_2,\ldots,b_{l-1}\}$, and $A\cap B=\phi$. Let $r$ and $l$ be two positive integers, for each $i\in Z_l$, $a_i$ is adjacent to $b_j$ if and only if $j\in\{i,i+1,i+2,\ldots,i+r-1\}$ $(\bmod l)$ then $G=(A,B)$ is called an $(r,l)-$crown is said to be even-degree if $r$ is even. Otherwise, it is odd-degree.

In this thesis, we prove that for each even integer $r\ge 6$, an even-degree $(r,2r-4)-$crown has an $\alpha$-labeling.

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# A Study on List Coloring Problems

**劉旭玲** ( Hsu-Ling Liu )

Department of Computer Science and Information Engineering
National Chi Nan University

**ADVISOR: 阮夙姿 教授**

## Abstract

List coloring is to color each vertex $v$ of graph $G$ from its color set $L(v)$. If any two adjacent vertices have different colors, $G$ is colored properly. We are interested in the smallest size of $L(v)$ for every vertex v such that each $L(v)$ has the same size and the graph $G$ is colored properly. And, the smallest size is called the list chromatic number of $G$. In this thesis, we discuss the list coloring problem for several graphs. First, we give an lower and upper bounds of the list chromatic number for cograph, which was unknown before. Secondly, we focus on the join operation of two paths, $P_m \vee P_n$. There are some results for the list chromatic number of two paths $P_m \vee P_n$. Finally, we study this problem on planar graph. We construct two planar graphs and show that the list chromatic numbers of them are not equal to 3.

## 中文摘要

列表著色是將圖 $G$ 上的每一點 $v$ 著上顏色，所著的顏色必須從 $v$ 所對應的顏色集合 $L(v)$ 中選取。若任意兩個有邊相連的點皆著上不同的顏色，則稱圖 $G$ 為可正當列表著色。當每個點的顏色集合大小皆相同並且圖 $G$ 為可正當列表著色時，去判斷最小可能之點的顏色集合大小即是我們所感興趣的。而此最小可能之點的顏色集合大小稱作圖 $G$ 的列表著色數。在本篇論文中，我們在數種圖形上討論列表著色問題。首先，我們提出了聯合圖形(cograph)的列表著色數之上、下界值。事實上，聯合圖形的列表著色數，在此之前尚未被討論過的。其次，我們集中心力在經過合併運算(join)的兩個路徑圖(path)上。在此部份，我們提出一些方法，根據這些結果可以對某些正整數 $m, n$ 判定 $P_m \vee P_n$ 的列表著色數。另一方面，我們依不同的條件，建立兩個平面圖並證明這兩個圖形的列表著色數皆不等於 3。

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# $(p, 1)$-total Labeling of Complete Graphs

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## Abstract

Given a graph $G$ and positive integers $p, q$ with $p \geq q$, the $(p,q)$-total number $\lambda^T_{p,q}(G)$ of $G$ is the width of the smallest range of integers that suffices to label the vertices and the edges of $G$ such that the labels of any two adjacent vertices are at least $q$ apart, the labels of any two adjacent edges are at least $q$ apart, and the difference between the labels of a vertex and its incident edges is at least $p$. Havet and Yu [7] first introduced this problem and determined the exact value of $\lambda^T_p(K_n)$ except for even $n$ with $p+5 \leq n \leq 6p^2-10p+4$. Their proof for showing that $\lambda^T_p(K_n) \leq n+2p-3$ for odd $n$ has some mistakes. In this thesis, we prove that if $n$ is odd, then $\lambda^T_p(K_n) \leq n+2p-3$ if $p=2, p=3$, or $4\left\lfloor \frac{p}{2} \right\rfloor + 2 \leq n \leq 4p-1$, and we extend some results that were given in [7]. Beside these, we also consider the lower bound of $\lambda^T_{p,q}(K_n)$ under the condition that $q < p < 2q$.

## 中文摘要

給定一個圖形$G$與正整數$p$、$q$，其中$p \geq q$，$G$的$(p,q)$全標號值$\lambda^T_{p,q}(G)$，是指在$G$中頂點與邊的標號值，其所使用的整數範圍之最小值，使得任意相鄰的兩頂點標號值至少差$q$，任意相鄰的兩個邊標號值至少差$q$，而頂點與其相連接的邊標號值至少差$p$。

Havet 與 Yu [7] 他們介紹了這一類問題，也計算了$\lambda^T_p(K_n)$切確值，但其中不包含$n$是偶數，且$p+5 \leq n \leq 6p^2-10p+4$之情況。另外他們在$n$是奇數，則$\lambda^T_p(K_n) \leq n+2p-3$ 的證明中有一些錯誤。在本論文中，我們證明了，當 $n$是奇數，若 $p=2, p=3$或$4\left\lfloor \frac{p}{2} \right\rfloor + 2 \leq n \leq 4p-1$，則$\lambda^T_p(K_n) \leq n+2p-3$；我們也延伸一些在[7]中的結果；除此之外，我們也考慮了在$q < p < 2q$的條件下， $\lambda^T_{p,q}(K_n)$的下界。

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# L(2,1) Labeling for *n*-Dimensional Graphs

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## Abstract

A $k-L(2,1)-$labeling $f$ for a given graph $G=(V,E)$, is a function $f:V\rightarrow\{0,1,2,\ldots,k\}$ such that $|f(x)-f(y)|\geq 2$ if $d(x,y)=1$, and $|f(x)-f(y)|\geq 1$ if $d(x,y)=2$ where $d(x,y)$ denotes the distance between vertices $x$ and $y$. The $L(2,1)-$labeling problem is finding the minimum $k$ such that $G$ has a $k-L(2,1)$ labeling. This paper deals with $L(2,1)-$labeling of star-like, pancake, and burnt pancake graphs.

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# Some Results on Distance-Two Labeling of a Graph

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## Abstract

For integer $n$ such that $n \geq 3$, a graph $G$ is called a generalized Petersen graph of order $n$ if and only if $G$ is a 3-regular graph consisting of two disjoint cycles (called inner and outer cycles) of length $n$, where each vertex of the outer (respectively, inner) cycle is adjacent to exactly one vertex of the inner (respectively, outer) cycle.

An $L^k(2,1)$-labeling of a graph G is a mapping f from the vertex set of $G$ to the set $\{0,1,2,\ldots,k\}$ such that $|f(x)-f(y)| \geq 2$ if $d(x,y)=1$ and $f(x) \neq f(y)$ if $d(x,y)=2$, where $d(x,y)$ is the distance between vertices $x$ and $y$ in $G$. The minimum $k$ for which $G$ admits an $L^k(2,1)$-labeling, denoted $\lambda(G)$, is called the $\lambda$-number of $G$.

Georges and Mauro [GM2002] conjectured that $\lambda(G) \leq 7$ for all generalized Petersen graphs $G$ of order $n \geq 7$. In 2006, Adams, Cass and Troxell [AT] proved that this conjecture is true for orders 7 and 8. In this paper we prove that Georges and Mauro's conjecture is true for order $n = 9$, 10, 11, and 12.

In 2004, Calamoneri and Petreschi considered $L(2,1)$-labeling on regular tilings of the plane. Because of the motivation by their results, we give the $\lambda$-number for the tiling of the plane which tiled by square and octagon and also a bound for the tiling of the plane which tiled by pentagon and heptagon.

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# On the Study of the Diameter of Chordal Ring Networks

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**Abstract**

One of the simplest topologies for interconnection networks is the ring network. Since the ring network has poor reliability and high transmission delay, hybrid topologies utilizing the ring network as a basis for synthesizing richer interconnection schemes have been proposed to improve the reliability and reduce the transmission delay (the diameter). The chordal ring network is a commonly used extension for the ring network and it is considered to be obtained by adding chords to a ring network so that the diameter can be reduced and the reliability can be increased. In [1], Arden and Lee proposed a formula for computing the diameter of a given chordal ring network; this diameter formula consists of three cases and case 3 of it consists of five subcases: subcases 3.1, 3.2, 3.3, 3.4 and 3.5. In [4], Huang and Chen pointed out that case 3 of the diameter formula is incorrect. In this thesis, we run a computer program to obtain the percentage of each of the three cases of the diameter formula, the percentage of each of the five subcases of case 3, and the faulty percentages for the five subcases of case 3. In particular, we observe that subcase 3.4 is never incorrect and the N that satisfying subcase 3.4 is of a specific form.

**中文摘要**

環狀網路是最簡單的網路架構。由於環狀網路的可靠度低、傳輸延遲高，因此，以環狀網路為基礎的混合式環狀網路結構相繼被提出，以提高其可靠度與降低其傳輸延遲(亦即直徑)。弦環式網路是在環狀網路的結構中增加弦，使得其可靠度提高、直徑降低。Arden 和 Lee 在[1]中對於弦環式網路發表了一個公式來計算其直徑；此直徑包含了三個狀況，且第三個狀況中還包含了五個子狀況：子狀況 3.1、3.2、3.3、3.4 和 3.5。Huang 和 Chen 在[8]中指出 Arden 和 Lee 的直徑公式中的第三個狀況是錯的。在本論文中，我們執行電腦程式以得到直徑公式中的每個狀況所佔有的百分比、和第三個狀況中五個子狀況錯誤的比例。我們特別觀察到直徑公式在子狀況 3.4 是完全正確的，而且符合子狀況 3.4 的 $N$ 有特殊形式。

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# On the All-to-All Personalized Exchange Problem in Chordal Ring Networks

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**Abstract**

In [5, 7], Masuyama et al. proposed two all-to-all communication algorithms for chordal ring networks of degree 3. The first algorithm (call it Algorithm A) is an all-to-all personalized exchange algorithm and it is used when there is no fault. The second algorithm (call it Algorithm B) is an all-to-all broadcast algorithm and it can tolerate one or two faults. In [5, 7], it has been proven that Algorithm A takes $\sum_{i=1}^{\frac{N}{2}} i$ time units to fulfill an all-to-all personalized exchange in a chordal ring network $CR(N,w)$, where $N$ is the number of nodes and w is the chord length of the chordal ring network. However, we observe that Algorithm A only utilizes ring-links to fulfill an all-to-all communication and Algorithm B utilizes chordal-links only when there are faults. Since all of the chordal-links are not used in any all-to-all communication when there is no fault, a huge amount of hardware is wasted. In this thesis, we will use chordal-links to facilitate an all-to-all personalized exchange. In particular, we propose an all-to-all personalized exchange algorithm that works for all chordal ring networks. We will show that our algorithm uses less time units to fulfill an all-to-all personalized exchange and hence improves Algorithm A. We also provide an all-to-all personalized exchange algorithm that works only for chordal ring networks with $w = 3$ and clarify some unclear parts and correct some incorrect parts in Algorithm B.

On the All-to-All Personalized Exchange Problem in Chordal Ring Networks

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## 中文摘要

弦環式網路中的邊可分為「弦邊」和「環邊」。在文獻[5]和文獻[7]中，Masuyama等學者提出了兩個弦環式網路的全體對全體通訊演算法，其中的第一個演算法（為方便，稱之演算法A）是一個全體對全體私有訊息傳送演算法，它使用於網路中沒有任何壞點時，第二個演算法（為方便，稱之演算法B）是一個全體對全體廣播演算法，當網路中有一至二個壞點時，它仍可使用。文獻[5]和文獻[7]證明了：對弦環式網路$CR(N,w)$而言，演算法A需要$\sum_{i=1}^{\frac{N}{2}} i$單位時間，其中$N$ 表示節點數，$w$ 表示弦長。然而，我們發現演算法A 只使用弦環式網路中的環邊來傳送訊息，演算法B 只在網路中有壞點時才會使用弦邊來傳送訊息。文獻[5]和文獻[7]的演算法浪費了大量的網路硬體，因為只要沒有壞點就不會使用弦邊。在這篇論文中，我們利用弦邊來使弦環式網路的全體對全體私有訊息傳送更快速。我們首先提出一個利用弦邊的演算法來執行弦環式網路的全體對全體私有訊息傳送；我們以實際數據來證明我們的演算法比演算法A 花費較少的單位時間，因此改進了演算法A。此外，我們也提出了一個針對$w = 3$ 的弦環式網路的全體對全體私有訊息傳送演算法，我們證明了此演算法比演算法A 花費少50%以上的單位時間。最後，我們闡明了演算法B 中的一些不清楚或不正確的地方。

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# On Antimagic Labeling of Graphs

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## Abstract

Let $G=(V,E)$ be an undirected graph with $p$ vertices and $q$ edges. Let $f$ be a bijection function from $E$ to $\{1,2,\ldots,q\}$. Then $f$ is an antimagic labeling of $G$ if the induced vertex sum $f^+:V\to Z^+$ is injective, where $f^+(u)=\sum_{uv\in E} f(u,v)$. If $G$ has an antimagic labeling then we said $G$ is antimagic. Furthermore, if there are integers $a,d$ such that $f^+$ is injective, and $f^+(V)=\{a,a+d,\ldots,a+(p-1)d\}$, then $f$ is an $(a,d)$-antimagic labeling of $G$. If $G$ has an $(a,d)$-antimagic labeling then we said $G$ is $(a,d)$-antimagic.

In this thesis, we obtain two results. First, we prove that all Cartesian products of star graphs with star graphs, $ST(m)\times ST(n)$, are anti-magic for integers $m\ge n\ge 3$. Finally, we discuss the conjecture that generalized Petersen graph $P(n,k)$ is $\left(\frac{5n+5}{2},2\right)$–antimagic for odd $n$, $n\ge 5$ and $2\le k\le \frac{n-1}{2}$.

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# A Study of Secret Image Sharing Schemes


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## Abstract

Secret image sharing scheme can efficiently solve the problems that are digit materials, military documents and important images could be forged, falsified and stolen, etc. Up to now, there are many scholars to design various secret image sharing schemes. However, most of the schemes exists many drawbacks and limits or still can be improved. For example: the computation is too complex; each participant holds too many shares; those schemes only be applied in specified access structure; cannot share multi-secret images, achieve the capabilities of the multi-use, verification and detection at the same time.

Based on the above defects, the goal of this thesis presents secret image sharing schemes with more integrity and multi-function. Therefore, we will divide thesis into three parts: In first part, Shyu and Chen proposed multiple secret images sharing schemes in 2008, but the secret images could be found by a set of unqualified participants in their general access structure sharing scheme. Therefore, we propose two mutli-secret images sharing schemes (*t, n*)-MSISS1 and GMSISS1. The sizes of the public image and time complexity of these two schemes are better than Shyu and Chen's schemes. Moreover, any set of the unqualified participants cannot found the secret images in these two schemes. In second part, Shi et al. proposed a (*t, n*)-threshold secret sharing scheme for image encryption in 2008, but there are some drawbacks in this scheme: the phenomenon shapes of the secret image could be found by using the public image, the width must be equal to the height, and the size of the secret images must be the same upon sharing multi-secret images. Therefore, we propose two multi-secret images sharing schemes (*t, n*)- MSISS2 and GMSISS2 to solve these problems. Furthermore, the proposed (*t, n*)-MSISS2 and GMSISS2 can respectively achieve sharing multi-secret images, multi-use, and each participant only holds one share, and the proposed GMSISS2 can be used for any general access structure. In third part, Zhao et al. proposed image secret sharing scheme to identify cheaters in 2009. But these exists the problem of disguise, and cannot achieve multi-use forever in Zhao et al.'s scheme. Therefore, we propose a verifiable and detectable multi-secret images sharing scheme with general access structure VDGMSISS to provide a verifiable procedure to authenticate identity of valid participants and a detectable method to check accuracy of the share. In addition, the proposed VDGMSISS can achieve some properties, such as sharing multi-secret images, multi-use, without security channel and be used for any access structure.

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## 中文摘要

秘密影像分享系統（Secret Image Sharing Scheme）可以有效地解決數位資料、軍事文件及重要影像被偽造、竄改及竊取等問題。迄今，已有許多學者建構各種秘密影像分享系統。然而，多數皆有其缺點與限制或可改進的空間，如：計算過於複雜、每位參與者所持有的部份秘密（Share）過多、只能運行於特殊的授權者集合（Access Structure）、無法同時分享多重機密影像、無法多次使用、無法達到可驗證與偵測欺騙者的能力等問題。

鑒於前述多項缺失，本篇論文的目的為提出更具完整性與多功能的秘密影像分享系統。此篇論文可分為三大部份：第一部份，在 2008 年徐等學者所提出的多重秘密影像分享系統中，針對一般授權者集合所設計的分享系統，其秘密影像可能會被一群不合法的參與者子集合中的所有參與者解出，導致系統的不安全。因此，我們提出二個改進的多重秘密影像分享系統 $(t,n)-$MSISS1 與 GMSISS1，在公佈影像的大小與時間複雜度上皆勝於徐等學者所提出的系統。同時，GMSISS1 使得不合法的參與者子集合無法解出不該解出的秘密影像。第二部份，在 2008 年施等學者提出 $(t,n)-$門檻值秘密影像分享系統。但在施等學著的系統中，任何人皆可能從公佈影像經簡單計算得到秘密影像的輪廓，而且此系統所分享的秘密影像之寬必須等於高。另外，施等學者的系統在分享多重秘密影像時，所有秘密影像的大小必須一致。因此我們提出了兩個多重秘密影像分享系統 $(t,n)-$MSISS2 與 GMSISS2；同時解決了洩密的問題及秘密影像的寬與高可彈性決定的問題。同時，$(t,n)-$MSISS2 與 GMSISS2 可達到分享多重秘密影像、多次使用，以及分享多重秘密影像時，每個參與者只持有一個部份秘密。其中 GMSISS2 可適用於一般授權者集合。第三部份，在 2009 年趙等學者提出可識別欺騙者之秘密影像分享系統，但在趙等學者的系統中，可能存在偽裝的問題及無法永久達到多次使用的功能。因此我們提出了一個針對一般授權者集合所設計之可驗證及偵測的多重秘密影像分享系統 VDGMSISS。不但提供了可驗證有效參與者的身分、與其所提出的部份秘密之正確性；此外，VDGMSISS 達到一些特性，諸如可分享多重秘密影像、可多次使用與無須存在秘密通道等。

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# An Extension of Stein-Lovász Theorem and Some of Its Applications

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## Abstract

The Stein-Lovász theorem provides an algorithmic way to deal with the existence of good coverings and then to derive some upper bounds related to some combinatorial structures. In order to deal with more combinatorial problems, an extension of the classical Stein- Lovász theorem, called the extended Stein- Lovász theorem, will be given in this thesis. Moreover, we will also discuss applications of the extended Stein- Lovász theorem to various models stated as follows:

1. Several disjunct matrices (for group testing purpose)
   - $d$-disjunct matrices, $(d;z]$-disjunct matrices;
   - $(d,r]$-disjunct matrices, $(d,r;z]$-disjunct matrices;
   - $(d,r)$-disjunct matrices, $(d,r;z]$-disjunct matrices;
   - $(d,$; $s$ out of $r]$-disjunct matrices, $(d$, $s$ out of $r$; $z]$-disjunct matrices.
2. Several selectors (for group testing purpose)
   - $(k,m,n)$-selectors, $(k,m,n;z)$-selectors;
   - $(k,m,c,n)$-selectors, $(k,m,c,n;z)$-selectors.
3. Some set systems (for others)
   - uniform $(m,t)$-splitting systems, uniform $(m,t;z)$-splitting systems;
   - uniform $(m,t_1,t_2)$-separating systems, uniform $(m,t_1,t_2;z)$-separating systems;
   - $(v,k,t)$-covering designs, $(v,k,t;z)$-covering designs;
   - $(v,k,t,p)$-lotto designs, $(v,k,t,p;z)$-lotto designs.

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# Forbidding Multicolored Cycles in an Edge-colored *Km,n*


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## Abstract

In an edge-colored graph, a subgraph whose edges are of distinct colors is known as a multicolored (or rainbow) subgraph. In this thesis, we shall first introduce several known results and conjectures related to multicolored subgraph in an edge-colored $K_n$, according to four categories of multicolored subgraphs. Then, we extend this study to consider whether there is a proper edge-coloring in a complete bipartite graph which forbids multicolored cycles. First, we claim that it is impossible to forbid multicolored 4-cycles in any proper $n$-edge-coloring of $K_{m,n}$ where $2 \le m \le n$ and $n \ge 4$. Second, we prove that any $n$-edge-colored $K_{m,n}$ $(m \le n)$ contains a multicolored $C_6$ if (*i*) $m \ge 3$ and $n \ge 9$; or (*ii*) $m \ge 4$ and $n = 7$. Finally, if $k$ is odd, we obtain a proper $2k$-edge-coloring of $K_{m,2k}$ which forbids multicolored ($2k$)-cycles where $k \le m \le 2k$.



## 中文摘要

在一個邊已著色的圖中，若有一個子圖它的每個邊的顏色皆不相同，我們稱這種子圖為混色子圖。在這篇論文中，我們先整理了一些以往有關混色子圖的定理與猜測，我們將依照子圖的種類分成四類來介紹；接下來我們討論在一個完全二部圖 $K_{m,n}$ 中，是否存在一種恰用了 $n$ 色的邊著色可以避免混色的圈出現，我們證明出來當 $2 \le m \le n$ 及 $n \ge 4$ 時，在 $K_{m,n}$ 中一定會產生混色的 $C_4$。而在下列兩種情形：(1) $m \ge 3$ 且 $n \ge 9$ 或(2) $m \ge 4$ 且 $n = 7$ 時，在 $K_{m,n}$ 中也會產生混色的 $C_6$。更進一步的，對於 $k \le m \le 2k$ 且 $k$ 為奇數時，我們找到一種 $2k$ 個顏色的著色法使得 $K_{m,2k}$ 中能避免混色的 $C_{2k}$ 出現。



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# The Multiplicity of Laplacian Eigenvalue One

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## Abstract

We give a tree algorithm of the multiplicity $m_T(1)$ of Laplacian eigenvalue 1. Let $T$ be the tree with a vertex $u$, and the vertices $w_1, w_2, \ldots, w_k, u_1, u_2, \ldots, u_s$ are all neighbors of $u$ with $\deg(w_i) = 1$ and $\deg(u_j) = 2$. For the remaining parts of $T, T_j$ is a tree with unique vertex $t_j$ in $T_j$ adjacent to $u_j$ for $1 \leq j \leq s$. Then

$$m_T(1) = (k-1) + \sum_{i=1}^{s} m_{T_i}(1).$$

In addition, we apply our algorithm to some special trees called caterpillar in our last section.

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# Characterizing the 2d-nary Banyan-type Networks

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**Abstract**

A Banyan-type network is a network that each input has a unique path to each output. A m-stage network is called a power-of-d network if for any stage-k and stage-l, the number of components in the subgraph of the relating underlying graph is a power of d. If a m-stage satisfies any two crossbars u, v in stage-k, its link to either the same crossbars or all distinct in stage-l, then it is called an universal buddy network. An m-stage network is a bit permutation network if the links of stage-k go from a group to another group in the next stage. Chang et al. (1999) characterized a part of the Banyan-type networks. Chen et al. (2005) also showed that an power-of-d universal buddy ( $d^p$ UB) network is equivalent to a bit permutation (BP) network. Hence we are interest in whether we could characterize the Banyan-type networks. Moreover, we also want to know whether it is possible to compute the number of the Banyan-type networks for a given m-stage network. Here come some problems when the stages become larger, the numbers of the Banyan-type networks increase fast. In this thesis, we give sufficient and necessary conditions for 2d-nary m-stage Banyan-type networks and some properties of Banyan-type to help us to compute the number of Banyan-type networks.

**中文摘要**

若一個網路的每個進線恰好有唯一一條路徑到每個出線，則稱此網路為一Banyan 型網路。在一個m 站式網路，對任意k 站至l 站，其無向子圖之component個數皆為d 的某次方，則此為一個power-of-d 網路. 對任意站內的任兩個交換器，其連接至另一站內交換器皆相同或完全相異，則此為一個universal buddy 網路. 一個位元排列網路則是任兩相鄰站內交換器之連線皆由一位元群至另一位元群. Chang et al. 在1999年首先提出位元排列網路刻劃了部份的Banyan 型網路同時也計算出它的數量，而Chen et al. 推廣到有額外站數的位元排列網路並且證明位元排列網路與power-of-d universal buddy 同構.

因此我們研究刻劃Banyan 型網路.

在這篇論文裡，我們刻劃了當d 是偶數時的Banyan 型網路，也證明了Banyan 型網路的充分必要條件，並且提出了一些Banyan 型網路的性質幫助計算Banyan 型網路的數量.

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# On Vertex Ranking and Related Problem of Graphs

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## Abstract

Given a connected graph $G$, a vertex ranking is a mapping $C:V \to \{1,2,\ldots,k\}$ such that for every path of end vertices $u,v$ and $C(u)=C(v)$, there is a vertex $w$ lies on the path with $C(w)>C(u)=C(v)$. The minimum value of $k$ among all vertices ranking of $G$ is called the vertex ranking number of $G$. For the on-line version, the vertices are given one by one in an arbitrary order together with the edges adjacent to the vertices that are already given. The rank of the vertex has to be assigned in real time and the rank cannot be changed after the assignment. Taking another point of view of the problem, there is only one vertex with the maximum rank, when removing this vertex, each component of the resulting graph still have only one vertex with the maximum rank in that component. Hence this problem may be extended to $c$-vertex ranking problem which consider each component may have no more than $c$ vertices with maximum rank in each level of vertex removing. The same problem may be considered on the strong digraphs. This thesis provided some polynomial time algorithms to determine the $c$-vertex ranking number for $c \geq 2$ of trees, wheels, complete bipartite graphs, and complete $r$-partite graphs in both offline and online versions; and discussed the maximum and minimum possible vertex ranking number for the strong orientation of those graphs.

## 中文摘要

對於一個給定的圖形$G$，點分級（vertex ranking）是將$G$ 上的每一個點，對應到一個正整數上，若存在任意兩個點對應到相同的正整數，則在這兩個點的所有路徑上，都存在至少一個對應到更大值的點。而對於圖形$G$ 所有點分級中最大編號的最小值即稱為這各圖形的點分級數（vertex ranking number）。若以即時的觀點來看，首先會知道這是什麼圖形並共有多少頂點，然後各頂點一個個隨機進入。每個頂點在進入後須隨即給予編號，而所有的資訊僅限於與之前進來點的相鄰狀況，並且編號值一但給定就不能再更改。若換各角度來看這各問題，可以視為在圖形中，只能有一個最大值的點，將此點拿掉之後，所存在的每個連通子圖同樣也只能存在一個最大值的點。相關的問題包括對於圖形中所存在的最大值的點不超過$c \geq 2$個時的點分級數還有在強連通有向圖上的點分級數問題等。本文中分別對於樹（tree）、車輪圖（wheel）、完全二分圖（complete bipartite）乃至於完全多分圖（complete r-partite），分別就一般和即時情況探討其點分級數及其相關問題。

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# S-Packing Coloring on Graphs

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## Abstract

The concept of the *S-packing coloring* motivates from the areas of frequency or channel assignment in wireless networks, resource placements and biological diversity. For instance, Federal Communications Commission of the United States Government has established numerous rules and regulations concerning the assignment of broadcast frequencies to radio stations. Two radio stations assigned the same broadcast frequency must be located sufficiently far apart so that the broadcast does not interfere with the reception of the other, and because of physical limitation, different frequency require different distance.

The *S-packing coloring* problem is defined as follows: given a finite non-decreasing sequence $(s_1, s_2, \ldots, s_k)$ of positive integers, a graph $G=(V,E)$ is called $(s_1, s_2, \ldots, s_k)$*-packing colorable* if there is a function $f: V \to \{1, 2, \ldots, k\}$ such that $d(x,y) > s_i$ or $x = y$ when $f(x) = f(y) = i$. For an infinite non-decreasing sequence $S = \{s_n\}_{n=1}^{\infty}$ of positive integers, the *S-chromatic number* $\chi_S(G)$ of $G$ is the minimum number $k$ such that $G$ is $(s_1, s_2, \ldots, s_k)$-packing colorable.

In this thesis, we find some sharp bounds of S-chromatic numbers of some classes of graphs. We also characterize graphs which attain the bounds. From a complexity point of view, we distinguish NP-completeness or P-solvablility for some S-packing coloring problems.

## 中文摘要

泛著寬色問題主要來自無線網路的頻道分配、資源配置及生物多樣性模型。舉例來說，為了避免互相干擾，美國聯邦通訊委員會規定廣播電台必須相隔足夠距離才能使用相同的頻率，因為物理的限制，不同頻率需要不同的距離，政府部門必須適當配置才能使整個廣播網路順利運作。

將此問題化約成圖論型式：對於給定的有限遞增正整數數列$(s_1, s_2, \ldots, s_k)$及圖$G$，若我們能夠給予圖上的每個頂點一個顏色，使得著第$i$色的兩個相異頂點距離超過$s_i$，則稱此圖可被$(s_1, s_2, \ldots, s_k)$著色；而對於給定的無限遞增正整數$S = \{s_n\}_{n=1}^{\infty}$，我們希望知道給定的圖最少要幾個顏色能夠作$(s_1, s_2, \ldots, s_k)$著色，此最少的數目稱為圖$G$的$S$-泛寬著色圖。

在這篇論文當中，我們找出某些類別圖的最佳上、下界或著色演算法、並刻劃某些達到泛寬著色上界的圖，在演算複雜度方面我們區別某些泛寬著色問題是複雜的非確定型演算法問題(NP-complete)或是多項式時間可解的問題。

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# A Study of DNA Graphs

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## Abstract

Molecular biology aims to study DNA and protein structure, that is the recognition of DNA primary structure. In order to do that, a mathematical model based on graph theory has been developed in recent years. Mainly, suitably defined digraphs are presented. A digraph built from the spectrum (a set of some k-long oligonucleotides) as follows: each oligonucleotide from the spectrum becomes a vertex, two vertices are connected by an arc if the $i$ rightmost nucleotides of the first point overlap with the $i$ leftmost nucleotides of the second one. We refer to these graphs as DNA graphs and DNA labelled graphs depending on whether the oligonucleotides used are distinct or not. In this thesis, we study the digraphs mentioned above and characterize DNA labelled graphs which are also DNA graphs, especially when the order (number of vertices) is small.

## 中文摘要

分子生物學主要是研究 DNA 序列及蛋白質的結構。由於序列的特性及讀取序列受到長度的限制，利用建構有向圖的數學模型可以有效地確定 DNA 序列及研究蛋白質的結構。此有向圖是這樣建構的：將每個長度為 $k$ 的核甘酸當成點，對於兩點 $x,y$，如果 $x$ 這點後 $i$ 段的核甘酸與 $y$ 這點前 $i$ 段的 DNA 序列要一樣，則 $x,y$ 有一條有向邊 $(x,y)$。我們將這類的圖稱作 DNA 圖或 DNA 標記圖。兩者的差別在於核甘酸是否有重複使用。在這邊論文中，我們主要是針對點數較小的圖去刻劃 DNA 圖的特性。

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# The Fibonacci Sequence in the Fibonacci Word

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## Abstract

There exists an infinite word on a 2-letters alphabet called: Fibonacci word. In this talk, I will first introduce the basics notions on the combinatorics of words, with many examples. Then I will present the so-called Fibonacci Word, explain where its name comes from and show how the Fibonacci sequence appeared in it in two different ways.

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# A Quadratic Algorithm for Finding Next-to-shortest Paths in Graphs

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## Abstract

Given an edge-weighted undirected graph $G$ and two prescribed vertices $u$ and $v$, a next-to-shortest $(u,v)$-path is a strictly-second shortest $(u,v)$-path amongst all $(u,v)$-paths with length strictly greater than the length of a shortest $(u,v)$-path. In this paper, we deal with the problem of computing a next-to-shortest $(u,v)$-path. We propose an $O(n^2)$ time algorithm for solving this problem, which significantly improves the bound of a previous one in $O(n^3)$ time where n is the number of vertices in $G$.

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# The Rigidity Property and the Unique Localization Problem of Sensor Networks

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## Abstract

In a sensor network, some nodes know their locations and other nodes determine their locations by measuring the distances to their neighbors. The process of computing the locations of the nodes is called network localization. A network localization problem is solvable if it has a unique solution. It has been proven in [1] that a network localization problem is solvable if and only if its corresponding grounded graph is globally rigid (i.e., 3-connected and redundantly rigid). A graph $G$ is redundantly rigid if $G - e$ is rigid for any edge e in $G$. In [5], Jacobs and Hendrickson have proposed an elegant algorithm to check if a given graph is rigid. In this thesis, we will provide several ways to construct rigid graphs from rigid graphs. We will also implement a computer program for solving the unique localization problem; in other words, our program can check if a given graph is globally rigid. Some experimental results will also be proposed.

## 中文摘要

在感應式網路中，有些點知道本身的所在位置，而其他的點經由計算它們與鄰居之間的距離去決定自己的所在位置，我們將計算這些點的所在位置的過程稱之為網路定位。如果一個網路定位問題有唯一解，則稱之為可被解決的。在文獻[1]中證明了網路定位問題是可被解決的，如果其對應的基礎圖是具有全範圍剛性性質(亦即三連通、且具有多餘的剛性性質）。在文獻[5]中，Jacobs 和Hendrickson提出了一個演算法來辨識一個給定的圖是否具有剛性性質。我們稱一個圖為具有多餘的剛性性質，假如我們移掉任何一個邊之後，此圖還具有剛性性質。在這篇論文中，我們將會提供數個從具有剛性性質的圖去建構一個新的具有剛性性質的圖的方法，我們也會提出一個電腦程式來解決唯一定位問題；換句話說，我們的程式可以判斷一個給定的圖是否具有全範圍剛性性質，我們也將會提出一些實驗的結果。

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# Constructing Independent Spanning Trees for Hypercubes and Locally Twisted Cubes

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## Abstract

The use of multiple independent spanning trees (ISTs) for data broadcasting in networks provides a number of advantages such as the increase of fault-tolerance and bandwidth. Thus the designs of multiple ISTs in several classes of networks have been widely investigated. In [27], Zehavi and Itai stated two versions of the $n$ independent spanning trees conjecture. The vertex (edge) conjecture is that any $n$-connected ($n$-edge-connected) graph has $n$ vertex-ISTs (edge-ISTs) rooted at an arbitrary vertex $r$. In [16], Khuller and Schieber proved that the vertex conjecture implies the edge conjecture. Recently, in [12], Hsieh and Tu proposed an algorithm to construct $n$ edge-ISTs rooted at vertex 0 for an $n$-dimensional locally twisted cube *LTQn*, which is a variant of the hypercube. Since *LTQn* is it not vertex-transitive, Hsieh and Tu's result does not solve the edge conjecture for the locally twisted cube. In the thesis, we con¯rm the vertex conjecture (and hence also the edge conjecture) for the locally twisted cube by proposing an algorithm to construct $n$ vertex-ISTs rooted at any vertex for the *LTQn*. We also con¯rm the vertex conjecture (and hence also the edge conjecture) for the hypercube.

## 中文摘要

在網路中使用多棵獨立擴張樹對於資料廣播有相當多的好處，例如：可以提高容錯以及頻寬等；因此，在各種的網路結構上，建造多棵獨立擴張樹，一直以來都被廣泛地研究。Zehavi 和Itai 在文獻[26]中，對於建造多棵獨立擴張樹提出了兩個猜測。「點猜測」闡述的是：在一個點連通度為$n$ 的圖上，能以圖中任一點為樹根，產生$n$ 棵點獨立擴張樹；「邊猜測」闡述的是：在一個邊連通度為$n$ 的圖上，能以圖中任一點為樹根，產生$n$ 棵邊獨立擴張樹。在文獻[16] 中，Khuller 和Schieber證明了點猜測能涵蓋邊猜測。局部扭轉超立方體是超立方體的變形。最近，Hsieh 和Tu 在文獻[10]中，提出了一個能在$n$維局部扭轉超立方體上，建造以0 為樹根的$n$ 棵邊獨立擴張樹的演算法。因為局部扭轉超立方體不具點對稱性質，Hsieh 和Tu 所提出的演算法無法解決局部扭轉超立方體的邊猜測。在這篇論文中，我們提出了一個可以在局部扭轉超立方體上，以任一點為樹根，建構$n$ 棵點獨立擴張樹的演算法；我們的演算法證明了局部扭轉超立方體符合點猜測，當然，也證明了局部扭轉超立方體符合邊猜測。此外，我們的演算法也能在超立方體上得到一樣的結果。

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# Mutually Independent Hamiltonian Cycles in *k*-ary *n*-cubes when *k* is odd

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## Abstract

The *k*-ary *n*-cubes, $Q_n^k$, is one of the most well-known interconnection networks in parallel computers. It has been shown that any $Q_n^k$ is a (2*n*)-regular, vertex symmetric graph with a Hamilton cycle. In this article, we will prove that any *k*-ary *n*-cubes, $Q_n^k$, contains 2*n* mutually independent Hamiltonian cycles, where $n \geq 2$ is an integer and $k \geq 3$ is an odd integer. More specifically, let $v_i \in Q_n^k$ for $0 \leq i \leq |Q_n^k|-1$ and let $\langle v_0, v_1, \ldots, v_{|Q_n^k|-1}, v_0 \rangle$ be a Hamilton cycle of $Q_n^k$. We prove that $Q_n^k$ contains 2*n* Hamilton cycles of the form $\langle v_0, v_1^l, \ldots, v_{|Q_n^k|-1}^l, v_0 \rangle$ for $0 \leq l \leq 2n-1$, in which $v_i^l \neq v_i^{l'}$ whenever $l \neq l'$. The result is optimal since each vertex of $Q_n^k$ has only 2*n* neighbors.

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# The Panpositonable Hamiltonicity of $k$-ary $n$-cubes


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## Abstract

The hypercube $Q_n$ is one of the most well-known and popular interconnection networks and the $k$-ary $n$-cubes $Q_n^k$ is an enlarged family from $Q_n$ that keeps many pleasing properties from hypercubes. However, as opposed to $Q_n$, $Q_n^k$ has not received enough attention. Hamiltonicity is important for the communication between processors in any interconnection network. The concept of hamiltonian panpositionability, proposed by S. Kao etc. [7], allows more flexible communication in a hamiltonian network. In this article, we study the panpositionable hamiltonicity of $Q_n^k$ if $k \geq 3$ is an odd integer, and the bipanpositionable hamiltonicity of $Q_n^k$ if $k \geq 4$ is an even integer.



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**2009 組合數學暨新苗研討會**

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