2012 Workshop on Graph Theory and Combinatorics & 2012 Symposium for Young Combinatorialists

2012 圖論和組合學研討會暨組合數學新苗研討會

August 10-12, 2012

Department of Applied Mathematics
National Sun Yat-sen University
Kaohsiung, Taiwan

http://www.math.nsysu.edu.tw/~comb/2012

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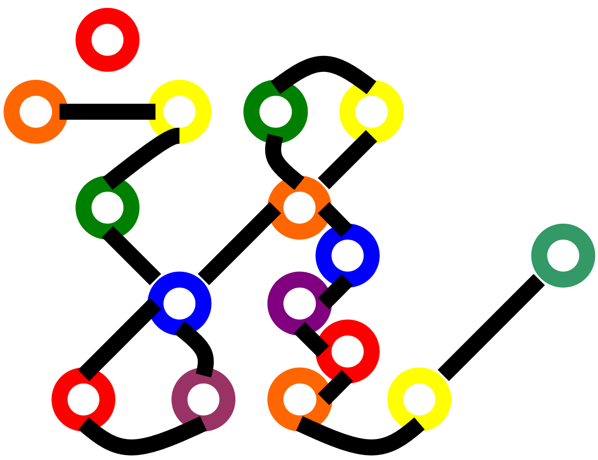

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大會宗旨(Purpose)

It is now a tradition of the combinatorics community in Taiwan to hold a Symposium for Young Combinatorialists in the summer, to provide a platform for the Ph.D. graduates and M.S. graduates of that year to report their research results, to communicate and discuss with each other. Also some senior professors are invited to give lectures at the symposium. It is an opportunity for discussion of mathematics, and also an opportunity to meet friends and make the society more like a big family. This event has been a tradition for more than twenty years now and has become an important event in the community. The symposium for the year 2012 will be held in National Sun Yat-sen University from August 10 to 12.

This year is special, as one of the founders of the combinatorics community in Taiwan, Professor Gerard J. Chang, turns 60. Gerard has made tremendous contribution to the development of combinatorics in Taiwan, through teaching and supervising students, doing administration work at university departments and at National Science Council (NSC), and pursuing research. To celebrate Gerard’s 60th birthday, we expand the scale of this annual symposium, by extending it to a 3 day workshop and inviting some scholars from abroad to this workshop.

As before, this symposium will issue Excellent Thesis awards at the end of the symposium to selected theses. The selection will be done in June and July, by a committee. Students graduating this year are encouraged to submit thesis to the organizing committee of the symposium.

組合數學新苗研討會旨在提供台灣剛取得碩博學位的同仁，發表其論文成果，互相切磋，並接受大家建議的機會，同時亦邀請幾位較資深的老師給予大會演講，增益研究上的學習與方向。

組合數學新苗研討會已連續舉辦有 20 餘年，且今年適逢張鎮華教授 60 歲，為感謝張老師對組合數學界的貢獻，我們擴大舉辦此次研討會。第一天為圖論及組合學研討會，以慶祝張老師六十歲生日為主，邀請國內外數位學者演講，共襄盛舉。第二日及第三日為組合數學新苗研討會，會中邀請幾位國內學者給予演講，剛取得碩、博士學位的新苗可於此時發表論文。此研討會對於鼓勵年青後起之輩與國內專家學者研究交流有很大的助益。

今年再度由中山大學應用數學系主辦。承續歷年的作法，今年亦將有最佳論文選拔，將組成公正的審查小組，在六、七月間審論文。獲選的同學，我們將在會議結束前頒發獎狀及獎牌。

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2012 圖論及組合學研討會暨組合數學新苗研討會**議程**

(**Conference Program**)

<u>地點(Place)</u>: 國立中山大學理學院國際會議廳

(**Conference Center,** College of Science**)**

<u>08 月 10 日 星期五 (August 10, Friday)</u>

09:30~10:00 報到(Registration)
10:00~10:10 開幕(Welcoming Remarks) 主持人(Chairman): Xuding Zhu

<u>Session 1</u>. 主持人(Chairman): Xuding Zhu
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(Page 02) 題目(Title): Cartesian product of oriented graphs with oriented hypercubes
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<u>Session 4</u>. 主持人(Chairman): Stephane Bessy
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(Page 07) 題目(Title): From topological methods to combinatorial proofs for Kneser graphs
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<u>Session 5</u>. 主持人(Chairman): Daphne Liu
15:45~16:35 邀請演講(Invited Speaker):葉鴻國 (Hong-Gwa Yeh, National Central University)
(Page 09) 題目(Title): Diffusion on networks
<u>Session 6</u>. 主持人(Chairman): Hong-Gwa Yeh
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Session 7. 邀請演講(Invited Talk) 主持人(Chairman): Chiuyuan Chen

09:00~09:50 邀請演講(Invited Speaker):游森棚 (Sen-Peng Eu, National University of Kaohsiung)

(Page 11) 題目(Title): Permutation patterns and ARM identities

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Session 8. 主持人(Chairman): Sen-Peng Eu

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(Page 13) 題目(Title): On the target set selection problem

10:45~11:10 演講者(Speaker):蔡維迦 (Wei-Chia Tsai (National University of Kaohsiung)

(Page 14) 題目(Title): Border strip decompositions on two-dimensional surfaces

11:15~11:40 演講者(Speaker):何澤初 (Tze-Chu Ho, National University of Kaohsiung)

(Page 15) 題目(Title): Bell permutation tableaux

11:50~13:00 午餐(Lunch)

Session 9. 主持人(Chairman): Sheng-Chyang Liaw

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(Page 16) 題目(Title): Relations between graphs

13:30~13:55 演講者(Speaker):劉純蓉 (Chun-Rong Liu, National Chiayi University)

(Page 17) 題目(Title): On the r-equitable coloring of complete bipartite graphs

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13:30~13:55 演講者(Speaker):高瑋琳 (Wei-Lin Kao, National Chiao Tung University)
(Page 29) 題目(Title): Facebook -- a smaller world
14:00~14:25 演講者(Speaker):李姿慧 (Zi-Hui Lee, National Chiao Tung University)
(Page 30) 題目(Title): A mathematical model for finding the culprit who spreads rumors
14:30~14:55 演講者(Speaker):劉晉宇 (National Sun Yat-sen University)
(Page 31) 題目(Title): The game Grundy arboricity of graphs
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Session 14. 主持人(Chairman): Xuding Zhu
15:10~ **頒發優良論文獎**
賦歸(End)

Families of Subsets with a Forbidden Subposet

Jerrold R. Griggs*

Abstract

Given a finite poset P, we consider the largest size $\mathrm{La}(n, P)$ of a family of subsets of $[n] := \{1, \ldots, n\}$ that contains no (weak) subposet P. Letting P_k denote the k-element chain (path poset), Sperner's Theorem (1928) gives that the largest size of an antichain of subsets of $[n]$, $\mathrm{La}(n, P_2) = \binom{n}{\lfloor n/2 \rfloor}$, and Erdős (1945) showed more generally that $\mathrm{La}(n, P_k)$ is the sum of the k middle binomial coefficients in n.

In recent years Katona and his collaborators investigated $\mathrm{La}(n, P)$ for other posets P. It can be very challenging, even for small posets. Based on results we have, Griggs and Linyuan Lu conjecture that $\pi(P) := \lim_{n\to\infty} \mathrm{La}(n, P)/\binom{n}{\lfloor n/2 \rfloor}$ exists for general posets P, and, moreover, it is an integer obtained in a specific way.

For $k \geq 2$ let D_k denote the k-diamond poset $\{A < B_1, \ldots, B_k < C\}$. Using probabilistic reasoning to bound the average number of times a random full chain meets a P-free family $\mathcal{F}$, called the Lubell function of $\mathcal{F}$, Griggs, Wei-Tian Li, and Lu prove that $\pi(D_2) < 2.273$, if it exists. This is a stubborn open problem, since we expect $\pi(D_2) = 2$. It is then surprising that, with appropriate partitions of the set of full chains, we can explicitly determine $\pi(D_k)$ for infinitely many values of k, and, moreover, describe the extremal D_k-free families. For these fortunate values of k, and for a growing collection of other posets P, we have that $\mathrm{La}(n, P)$ is a sum of middle binomial coefficients in n, while for other values of k and for most P, it seems that $\mathrm{La}(n, P)$ is far more complicated.

Some techniques being used are adapted from Turán theory of graphs and hypergraphs, including probabilistic arguments and, more recently, flag algebras.

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Cartesian Product of Oriented Graphs with Oriented Hypercubes

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Introduction

Let $G = (V, E)$ be a finite simple undirected graph of order n with $V = \{v_1, v_2, \dots, v_n\}$ as its vertex set and E as its edge set. Let σ be any orientation of the edge set E yielding the oriented graph $G^\sigma = (V, \Gamma)$, where Γ is the arc set of G^σ. The adjacency matrix of G is the $n \times n$ matrix $A = (a_{ij})$, where $a_{ij} = 1$ if $(v_i, v_j) \in E$ and $a_{ij} = 0$ otherwise. As the matrix A is real and symmetric, all its eigenvalues are real. The skew adjacency matrix of the oriented graph G^σ is the $n \times n$ matrix $S(G^\sigma) = (s_{ij})$, where $s_{ij} = 1 = -s_{ji}$ whenever $(v_i, v_j) \in \Gamma(G^\sigma)$ and $s_{ij} = 0$ otherwise. Clearly $S(G^\sigma)$ is a skew symmetric matrix and hence its eigenvalues are all pure imaginary. The spectrum of G, denoted by $Sp(G)$, is the set of all eigenvalues of A. In a similar manner, the skew spectrum of the oriented graph G^σ is defined as the spectrum of the matrix $S(G^\sigma)$.

Cartesian product of oriented graphs with oriented hypercubes

Suppose H^σ is an oriented graph of order n with H as its underlying graph. For $d \geq 1$, let $G_d = H \Box Q_d \simeq Q_d \Box H$, where $Q_d = K_2 \Box K_2 \Box \dots \Box K_2$ (d times), be the Cartesian product of the undirected graph H with the hypercube Q_d of dimension d. We construct an oriented Cartesian product graph G_d^ψ by orienting the edges of G_d in a specific way. The skew adjacency matrices $S(G_d^\psi)$ obtained in this way for some special families of G answer some special cases of the Inverse Eigenvalue Problem. The Cartesian product graph $G_d = Q_d \Box H = K_2 \Box (Q_{d-1} \Box H) = K_2 \Box G_{d-1}$ contains two copies of G_{d-1}. Set $G_0^\psi \simeq H^\sigma$. Figure 1 gives the way of orienting the graph G_{k+1}^ψ from two copies of G_k^ψ, for $k = 1, 2, \dots$

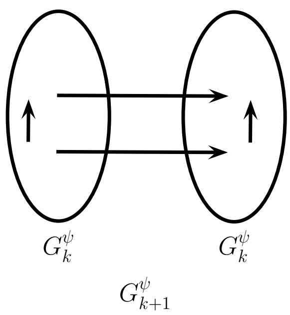

Figure 1: Construction of G_{k+1}^ψ from two copies of G_k^ψ

For $d \geq 1$, let S_d be the skew adjacency matrix of the Cartesian product graph G_d^ψ oriented as defined above. Then the skew adjacency matrix S_{d+1} for the oriented graph G_{d+1}^ψ is given by

$$S_{d+1} = \begin{bmatrix} S_d & I \\ -I & S_d \end{bmatrix},$$

where I is the identity matrix of order 2^d.

Theorem 1. *For $d \geq 1$, the skew spectrum of the oriented graph G_{d+1}^ψ is*

$$Sp_S(G_{d+1}^\psi) = \{\mathbf{i}(\mu \pm 1) \ : \ \mathbf{i}\mu \in Sp_S(G_d^\psi)\}.$$

Lemma 2 ([2])**.** *Let $Sp(G_1) = \{\lambda_1, \ldots, \lambda_n\}$ and $Sp(G_2) = \{\mu_1, \ldots, \mu_t\}$ be respectively the adjacency spectra of two graphs G_1 of order n and G_2 of order t. Then*

$$Sp(G_1 \Box G_2) = \{\lambda_i + \mu_j \ : \ 1 \leq i \leq n, \ 1 \leq j \leq t\}.$$

Corollary 3. *Suppose σ is an orientation of a bipartite graph H for which $Sp_S(H^\sigma) = \mathbf{i}Sp(H)$. Then for each $d \geq 1$, the oriented Cartesian product graph $G_d^\psi = (H \Box Q_d)^\psi$ has the property that*

$$Sp_S(G_d^\psi) = \mathbf{i}Sp(G_d).$$

It is easy to verify that when $H^\sigma \simeq K_2^\sigma$, the oriented graph $G_d^\psi = (H \Box Q_d)^\psi \simeq Q_{d+1}^\psi$ is one of the oriented hypercubes constructed by G-X. Tian (see Algorithm 2 in [7]) for which $Sp_S(G_d^\psi) = \mathbf{i}Sp(G_d)$.

Theorem 4. *Let $Sp_S(H^\sigma) = \begin{pmatrix} \mathbf{i}\mu_1 & \mathbf{i}\mu_2 & \ldots & \mathbf{i}\mu_p \\ m_1 & m_2 & \ldots & m_p \end{pmatrix}$, where m_j, $1 \leq j \leq p$, is the multiplicity of $\mathbf{i}\mu_j$. Then the skew spectrum of the oriented graph $G_d^\psi = (H \Box Q_d)^\psi$ is given by*

$$Sp_S(G_d^\psi) = \begin{pmatrix} \mathbf{i}(\mu_j + d) & \mathbf{i}(\mu_j + d - 2) & \ldots & \mathbf{i}(\mu_j + d - 2d) \\ \binom{d}{0} m_j & \binom{d}{1} m_j & \ldots & \binom{d}{d} m_j \end{pmatrix}, \ \ j = 1, 2, \ldots, p.$$

Definition 5. *Suppose G^σ is an oriented graph with skew spectrum $Sp_S(G^\sigma) = \begin{pmatrix} \mathbf{i}\mu_1 & \mathbf{i}\mu_2 & \ldots & \mathbf{i}\mu_p \\ m_1 & m_2 & \ldots & m_p \end{pmatrix}$. Then G^σ is said to be skew integral if each μ_j, $1 \leq j \leq p$, is an integer.*

The following corollary constructs many skew integral oriented graphs from a given skew integral oriented graph.

Corollary 6. *Let H^σ be a skew integral oriented graph. Then for each $d = 1, 2, \ldots,$ the oriented Cartesian product graph $G_d^\psi = (H \Box Q_d)^\psi$ is also skew integral.*

Structured Inverse Eigenvalue Problem (SIEP)

Given a certain spectral data, the objective of a Structured Inverse Eigenvalue Problem (SIEP) is to construct a matrix that maintains a certain specific structure as well as the given spectral property [5]. Structured Inverse Eigenvalue Problems arise in a wide variety of fields: control design, system identification, principal component analysis, exploration and remote sensing, geophysics, molecular spectroscopy, particle physics, structure analysis, circuit theory, mechanical system simulation and so on [5]. For further study on inverse eigenvalue problems, the reader may refer to the survey papers [3, 5]. The SIEP is defined as follows:
Structured Inverse Eigenvalue Problem (*SIEP*)*:* Given scalars $\lambda_1, \lambda_2, \ldots, \lambda_n$ in a field $\mathbb{F}$, find a specially structured matrix A such that $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ forms the set of eigenvalues of A.

We now construct skew symmetric matrices that solve the SIEP for some interesting special sets of complex numbers.

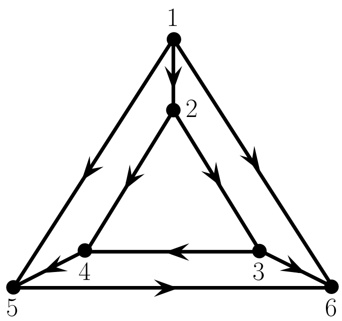

Figure 2: Oriented graph H^σ

Theorem 7. *Let H^σ be the oriented graph given in Figure 2. Let $\mathscr{G} = \{G_d^\psi \ : \ d = 1, 2, \ldots\}$ be the family of oriented graphs obtained by orienting $G_d = H\Box Q_d$ as shown in Figure 1. Then the oriented graph $G_d^\psi = (H\Box Q_d)^\psi$, $d = 0, 1, 2, \ldots,$ of order $n_d = 6.2^d$ is skew integral.*

Theorem 8. *Given a positive integer $k \geq 3$, there exists a skew symmetric matrix A such that the set $\{0, \pm\mathbf{i}, \pm 2\mathbf{i}, \ldots, \pm k\mathbf{i}\}$ forms the set of all distinct eigenvalues of A.*

By a cycle in an oriented graph G^σ we mean a cycle, not necessarily directed. An even cycle C of G^σ is said to be *evenly oriented* (respectively *oddly oriented*) if the number of arcs of C in each direction is even (respectively odd) [6]. From [1], we know that the skew adjacency matrices of both the evenly oriented cycles C_4^σ have the same skew spectrum, namely,

$$Sp_S(C_4^\sigma) = \begin{pmatrix} 2\mathbf{i} & 0 & -2\mathbf{i} \\ 1 & 2 & 1 \end{pmatrix}.$$

Applying our orientation technique (as shown in Figure 1) to an evenly oriented cycle C_4^{σ}, we can construct oriented graphs $G_d^{\psi} = (C_4 \square Q_d)^{\psi}$, $d = 1, 2, \ldots$, with the following special property:

Theorem 9. *For any $d \in \mathbb{N}$, there exists a skew symmetric matrix A for which the set $\{\pm d\mathbf{i}, \pm(d-2)\mathbf{i}, \pm(d-4)\mathbf{i}, \ldots, \pm(d-k)\mathbf{i}\}$, where $k = d$ or $d-1$ according as d is even or odd, forms the set of all distinct eigenvalues of A.*

Orientation of hypercubes

Finally we present a new orientation ϕ to the hypercube Q_d for which the skew energy equals the energy of the underlying hypercube, distinct from the two orientations of hypercubes defined in [7].

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Enumerating the edge-colourings and total colourings of a regular graph*

S. Bessy† and F. Havet‡

Motivated by some algorithmic considerations, we are interested in computing the number of edge colourings of a connected graph. Precisely, we prove that the maximum number of k-edge-colourings of a connected k-regular graph on n vertices is $k \cdot ((k-1)!)^{n/2}$. Our proof is constructive and leads to a branching algorithm enumerating all the k-edge-colourings of a connected k-regular graph in time $O^*(((k-1)!)^{n/2})$ and polynomial space. In particular, we obtain a algorithm to enumerate all the 3-edge-colourings of a connected cubic graph in time $O^*(2^{n/2}) = O^*(1.4143^n)$ and polynomial space. This improves the running time of $O^*(1.5423^n)$ of the algorithm due to Golovach, Kratsh and Couturier [*WG10*].
In this talk, I will present our work and overview the known results on computing aspect of graph coloring.

*This work is supported by the French *Agence Nationale de la Recherche* under reference AGAPE ANR-09-BLAN-0159.

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From Topological Methods to Combinatorial Proofs for Kneser Graphs

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Abstract

Lovász [6] in 1978 confirmed the Kneser conjecture that the chromatic number of the Kneser graph $\mathrm{KG}(n,k)$ is equal to $n-2k+2$. Since then, algebraic topology has became an important tool in solving problems in combinatorics. In particular, various alternative proofs and generalizations of the Lovász theorem have been developed (cf. [1, 2, 7, 11]). Most of these proofs utilized methods or results in algebraic topology, mainly the Borsuk-Ulam theorem and its extensions.

On the other hand, combinatorial proofs for these remarkable results were also established. In 2004, using Tucker's lemma [10], Matoušek [8] presented the first combinatorial proof for the Lovász theorem, without any topological terms. In addition, Ziegler [11] gave combinatorial proofs for various generalizations of the Kneser coloring theorem.

Chen [4] in 2011 confirmed the Johnson-Holroyd-Stahl conjecture that the circular chromatic number of $\mathrm{KG}(n,k)$ also equals to $n-2k+2$. A shorter proof of this result was given by Chang, Liu, and Zhu [3]. Both proofs were based on Fan's lemma [5] in algebraic topology.

We present a simple self-contained combinatorial proof for Chen's theorem, without topological terms. The method is by specializing a constructive proof [9] of Fan's lemma and using the labeling function in [3]. Our approach also gives another simple self-contained combinatorial proof for the Lovász Theorem.

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Diffusion on Networks

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In this talk, we survey the combinatorial models of several existing diffusion problems over networks in areas such as economics, epidemiology, mathematics, politic and computer science.

Keywords: Spread of social influence, propagation model, viral marketing.

On-line list colouring of graphs

Xuding Zhu

Abstract

Given a graph G and a mapping $f : V(G) \to \{1, 2, \ldots, \}$. The f-list colouring game is played as follows: There are two players: A and B. Initially, all vertices are uncoloured. The integer $f(v)$ is the number of permissible colours v can receive. At the ith round, Player A selects a nonempty subset X_i of uncoloured vertices of G. This is the set of vertices for which i is a permissible colour. Player B selects an independent subset I_i contained in X_i. This is the set of vertices coloured with colour i. After this move, vertices in I_i are coloured. The game ends if every uncoloured vertex v has occurred in $f(v)$ of the X_i's, i.e., has been given $f(v)$ permissible colours. Player B wins the game if at the end of the game, all vertices are coloured. Otherwise Player A wins the game. The on-line choice number of G is the minimum intger k such that for the constant mapping $f(v) = k$ for all v, Player B has a winning strategy in the f-list colouring game. In this talk, I will survey some results on the study of on-line choice number of graphs.

Permutation Patterns and ARM identities

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After the seminal work of Schmidt and Simion in 1985, the booming results in 2000s and the proof of Stanley-Wilf conjecture by Tardos-Marcus in 2005, permutation patterns nowadays is still a very active research topic in combinatorics with many important problems left unanswered. In the first part of this expository talk we will give a quick survey of the history, the core results and the trends on this subject. In the second half of the talk we will introduce the concept of ARM-type identities and give several examples. These examples are joint separately with Fu, Pan, Ting, and Yan.

Keywords: Permutation patterns, ARM identity.

The game $L(d,1)$-labeling problem of graphs

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Let G be a graph and let k be a positive integer. Consider the following two-person game which is played on G: Alice and Bob alternate turns. A move consists of selecting an unlabeled vertex v of G and assigning it a number a from $\{0, 1, 2, \cdots, k\}$ satisfying the condition that, for all $u \in V(G)$, u is labeled by the number b previously, if $d(u, v) = 1$, then $|a - b| \geq d$, and if $d(u, v) = 2$, then $|a-b| \geq 1$. Alice wins if all the vertices of G are successfully labeled. Bob wins if an impasse is reached before all vertices in the graph are labeled. The *game* $L(d,1)$*-labeling number* of a graph G is the least k for which Alice has a winning strategy. We use $\tilde{\lambda}_1^d(G)$ to denote the game $L(d,1)$-labeling number of G in the game Alice play first, and use $\tilde{\lambda}_2^d(G)$ to denote the game $L(d,1)$-labeling number of G in the game Bob play first. In this talk, some results concerning this new problem will be introduced.

On the target set selection problem

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(Advisor: Hong-Gwa Yeh)

Consider the following hypothetical scenario as a motivating example. A company wish to market a new product. The company has at hand a description of the social network G formed among a sample of potential customers, where the vertices represent customers and edges connect people to their friends. The company wants to target key potential customers S of the social network and persuade them into adopting the new product by handing out free samples. We assume that individuals in S will be convinced to adopt the new product after they receive a free sample, and the friends of customers in S would be persuaded into buying the new product, which in turn will recommend the product to other friends. The company hopes that by word-of-mouth effects, convinced vertices in S can trigger a cascade of further adoptions, and finally all potential customers will be persuaded to buy the product.

A social network (G, θ) is usually modeled by a graph G together with a threshold function $\theta : V(G) \to \mathbb{N}$ such that $1 \leq \theta(v) \leq d_G(v)$ for each vertex v in G. Given a vertex subset S of a connected social network (G, θ). Consider the following repetitive process played on (G, θ). At round 0 (the beginning of the game), the vertices of S are colored black and the other vertices are colored white. After that, at each round $t > 0$, all white vertices v that have at least $\theta(v)$ black neighbors at the previous round $t-1$ are colored black. The colors of the other vertices do not change.

The process runs until no more white vertices can update colors from white to black. The set S is called a *target set* for (G, θ). We are interested in the following optimization problem: finding a target set S of smallest possible size such that all vertices of $V(G) \setminus S$ are black in the end.

Keywords: social networks, diffusion, viral marketing, influence spreading, target set selection.

Border Strip Decompositions On Two-Dimensional Surfaces

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We enumerate the border strip decompositions of a rectangular Young diagram which is generalized in a manner similar to the topology of two-dimensional surfaces such as a cylinder, Möbius band, torus, Klein bottle, and projective plane, by identifying pairs of opposite edges of the diagram.

We also enumerate fixed points of border strip decompositions on the surfaces under rotations, where a border strip decomposition is a fixed point if it is invariant under the operations.

Keywords: border strip decompositions, two-dimensional surfaces, fixed points.

Bell Permutation Tableaux

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The concept of permutation tableaux was introduced by Postnikov in the context of enumeration of the totally positive Grassmannian cells. It is known that the number of permutation tableaux of length n is $n!$. Corteel and Nadeau gave a bijection ϕ between the set of permutation tableaux and the set of permutations, and introduced L-Bell and R-Bell tableaux, both counted by the Bell numbers, as two subclasses of the permutation tableaux. Chen and Liu then characterized the corresponding permutations of the L-Bell tableaux under the bijection ϕ.

In this thesis we introduce three new subclasses of permutation tableaux, namely L'-Bell, R'-Bell, and B-Bell tableaux, and prove that they are also counted by the Bell numbers. We give characterizations of R-Bell and B-Bell tableaux in terms of pattern-avoiding permutations under the bijection ϕ.

We also introduce statistics on these five subclasses of tableaux and prove that they are equidistributed with the p, q-Stirling numbers of Wach and White on the 01-tableaux of Leroux. Meanwhile a new bijection consistent with the above q-statistic, but different from Corteel and Nadeau's, is given between L-Bell tableaux and R-Bell tableaux.

We also investigate the cardinality of the intersection of two of these subclasses. It turns out that many familiar classical sequences appear. Among them we prove that the cardinality of the intersection of the L-Bell and R-Bell tableaux (of the same length) is a Bessel number.

Keywords: Permutation, Permutation tableaux, Bell numbers, 01-tableaux, p, q-stirling numbers, L-Bell, R-Bell, B-Bell, L'-Bell, R'-Bell, Bessel numbers.

Relations Between Graphs

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Given two graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, we ask under which conditions there is a relation $R \subseteq V_G \times V_H$ that generates the edges of H given the structure of graph G. This construction can be seen as a form of multihomomorphism. It generalizes surjective homomorphisms of graphs and naturally leads to notions of R-retractions, R-cores, and R-cocores of graphs. Both R-cores and R-cocores of graphs are unique up to isomorphism and can be computed in polynomial time.

Keywords: generalized surjective graph homomorphism, R-reduced graph, R-retraction, binary relation, multihomomorphism, R-core, cocore.

On the r-Equitable Coloring of Complete Bipartite Graphs

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A graph G consists of a nonempty vertex set $V(G)$ and an edge set $E(G)$. Let $k \geq 1$ be an integer. A **(proper) k-coloring** of a graph G is a mapping $f : V(G) \rightarrow \{1, 2, \ldots, k\}$ such that adjacent vertices have different images. The images are called **colors** and all vertices of a fixed color constitute a **color class**. Then a k-coloring of a graph G is said to be **r-equitable** for any $r \geq 0$ if the size of any two color classes differ by at most r. And, a graph G is called **r-equitably k-colorable** if G has an r-equitable k-coloring. Besides, the least k such that a graph G is r-equitably k-colorable is called the **r-equitable chromatic number** of G and denoted $\chi_{r=}(G)$. Also, the least n such that a graph G is r-equitably k-colorable for all $k \geq n$ is called the **r-equitable chromatic threshold** of G and denoted $\chi^*_{r=}(G)$. In fact, the notion of **r-equitable colorability** is a natural generalization of the well-known **equitable colorability**, which is the case when $r = 1$.

A graph G is called a **bipartite graph**, denoted by $G(X, Y)$, if $V(G)$ can be partitioned into two subsets X and Y such that every edge of G joins a vertex of X to a vertex of Y. Moreover, if every vertex of X is adjacent to every vertex of Y, then we call $G(X, Y)$ a **complete bipartite graph**. Besides, if the sizes of X and Y are s and t, respectively, then a complete bipartite graph $G(X, Y)$ is denoted by $K_{s,t}$. When both s and t are equal to some positive integer n, $K_{s,t} = K_{n,n}$ is also called a **balanced complete bipartite graph**.

In this thesis, we first propose a necessary and sufficient condition for a (balanced) complete bipartite graph to be r-equitably k-colorable. Then we derive explicit formulas related to the r-equitable chromatic number and the r-equitable chromatic threshold of a (balanced) complete bipartite graph. Finally, we have some other results on the r-equitable k-coloring of a (balanced) complete bipartite graph.

Keywords: Equitable coloring; r-Equitable coloring; Complete bipartite graph; r-Equitable chromatic number; r-Equitable chromatic threshold.

The strong chromatic index of cacti

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A strong edge coloring of a graph G is an assignment of colors to the edges of G such that two distinct edges are colored differently if their distance is at most two. The strong chromatic index of a graph G, denoted by $\chi'_s(G)$, is the minimum number of colors needed for a strong edge coloring of G. For a graph G, define $\sigma(G) := \max_{uv \in E(G)} (d_G(u) + d_G(v) - 1)$, where $d_G(x)$ is the degree of x. A cactus is a connected graph in which every block is an edge or a cycle. In this thesis, we study strong chromatic edge coloring for cacti. In particular, it is proved that for any cactus G, we have $\chi'_s(G) = \sigma(G)$ if the length of any cycle is a multiple of 6; $\chi'_s(G) \leq \sigma(G) + 1$ if the length of any cycle is even; and $\chi'_s(G) \leq \lfloor \frac{3\sigma(G)-1}{2} \rfloor$ in general except the case $G = C_5$.

Keywords: Strong chromatic index, Cactus, Graph, Degree.

Four Partition Problems of Graphs

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We first study the first-fit chromatic numbers of graphs. Given a family $\mathcal{F}$ of graphs which is closed under taking induced subgraphs and $e(G) \leq dn(G)$ for any $G \in \mathcal{F}$, where $d > 0$ is fixed, we give an upper bound for the first-fit chromatic number of any graph in $\mathcal{F}$. This result applies to d-degenerate graphs, planar graphs, and outerplanar graphs.

We study the max-coloring problem of a vertex-weighted graph (G, c), which attempts to partition $V(G)$ into independent sets such that the sum of the maximum weight in each independent set is minimum. We give an upper bound for the number of sets needed in an optimal vertex partition of a vertex-weighted r-partite graph. We then derive the Nordhaus-Gaddum inequality for vertex-weighted graphs. We also consider the properties of the perfection on vertex-weighted graphs.

The balanced decomposition number $f(G)$ of a graph G is the minimum ℓ such that for any disjoint $R, B \subseteq V(G)$ with $|R| = |B|$ there is a partition $\mathcal{P}$ of $V(G)$ satisfying that $G[S]$ is connected, $|S \cap R| = |S \cap B|$ and $|S| \leq \ell$ for any $S \in \mathcal{P}$. We give a shorter proof of a known result that $f(G) = 3$ if and only if G is $\lfloor \frac{n(G)}{2} \rfloor$-connected and G is not a complete graph. We then extend the definition to k disjoint sets, and call the corresponding parameter the balanced k-decomposition number. We compute the balanced k-decomposition numbers of trees and complete multipartite graphs.

The parity (strong parity) edge-chromatic number of a graph G is the minimum number of colors used in an edge-coloring of G such that any path (open walk) of positive length uses some color an odd number of times. We prove that, for $3 \leq m \leq n$ and $n \equiv 0, -1, -2 \pmod{2^{\lceil \lg m \rceil}}$, the (strong) parity edge-chromatic number of $K_{m,n}$ is the least integer ℓ such that $\binom{\ell}{k}$ is even for each k with $\ell - n < k < m$. We also consider the parity and the strong parity edge-chromatic numbers of the products of graphs.

Keywords: first-fit chromatic numbers, max-coloring problem, balanced decomposition numbers, parity edge-colorings.

Lit-only sigma-game on nondegenerate graphs

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A configuration of the lit-only σ-game on a graph Γ is an assignment of one of two states, *on* or *off*, to each vertex of Γ. Given a configuration, a move of the lit-only σ-game on Γ allows the player to choose an *on* vertex s of Γ and change the states of all neighbors of s. In this talk, we introduce the development of the lit-only σ-game on finite simple graphs, especially nondegenerate graphs.

Keywords: Lit-only σ-game, group action, nondegenerate graph.

On the study of position-based routing algorithms for wireless ad hoc networks

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This thesis considers the problem of designing efficient position-based routing algorithms for wireless ad hoc networks. A routing algorithm is position-based if a node forwards its packet according to the position information (i.e., coordinates in the plane). GREEDY, COMPASS, ELLIPSOID, and FACE are four famous position-based routing algorithms. The former three algorithms run very fast but cannot guarantee message delivery. On the other hand, FACE does not run fast but it guarantees message delivery. It is indeed a challenge to develop an algorithm that can run very fast and can have high delivery rate at the same time. The purpose of this thesis is to propose two such algorithms. Experimental results show that our algorithms are quite good.

Keywords: wireless ad hoc network, position-based routing, delivery guarantee, path dilation, unit disk graph.

Computing Wide Diameters of Alternating Group Graphs

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A set of all even permutation over n-objects is called alternating group of degree n. Jwo, Lakshmivarahan and Dhall proposed alternating group graphs in 1993. An alternating group graph is a Cayley graph. In comparison with an n-dimensional star graph, an$n$-dimensional alternating group graph has half the number of vertices and approximately twice its degree. Alternating group graphs have some favorable properties such as low diameter, rich fault tolerance those are attractive for building massively parallel computers. Jwo, et al. presented that alternating group graphs are vertex symmetry, edge symmetry, hierarchical, hamiltonian, embeddability, and broadcastability. Accordingly, several researches investigated the topological properties of alternating group graphs. For the purposes of evaluating maximum parallelism, minimum transmission delay, reliability and fault tolerant ability, constructing vertex disjoint paths and determining wide diameters of interconnection networks are very important issues. Lin and Chiu (2002) proposed a routing scheme for constructing node-to-node disjoint paths in alternating group graphs. However, these paths constructed by Lin and Chiu may coincident on one node. Furthermore, Lin and Chiu have not yet computed wide diameters of alternating group graphs. This work derives a routing algorithm for constructing a container of width $2(n-2)$ between a pair of vertices in an alternating group graph with connectivity $2(n-2)$. Based on the provided algorithm, the wide diameter of an n-dimensional alternating group graph can be computed as its diameter plus 1 or 2.

Keywords: Alternating group graphs, fault tolerance, wide diameter, container, interconnection networks.

Visual Multi-Secret Image Sharing Scheme by Shifting Random Grids

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Visual cryptography (VC) is a visual secret sharing (VSS) method which is different from traditional cryptography. Visual Cryptography was proposed by Naor and Shamir in 1994. The technology combines traditional cryptography with conception of information sharing, encoding a secret image into n pieces of share images. We show the share images to participants who will obtain unidentified images which cannot be restored to original secret image when participants only have one share image. The main concept of VC is needless using computer and cryptographic computation when decode the secret images. What is needed is to collect k pieces shared images and superimposed them, then the original secret image can be distinguished, decoded, and reconstructed by human visual system.

The visual secret sharing scheme we proposed uses the conception of the random grid (RG). Recently, it has drawn more and more attention to encode more than one secret image into two share images by RG-based VSS techniques. But those researches can only encrypt at most four secret images in one time. And the distortion of the reconstructed secret images will be obvious when we increase the number of the secret images. In this thesis, we propose a RG-based scheme which encrypts multi-secret images into two shares by shifting random grids. Compared with the traditional VC-based VSS, RG-based VSS need not to design the codebook of conventional VC and the size of share images will not be expanded. Also, users can adjust the distortion in our schemes. The concept of our schemes in this thesis calculates each pixel of the secret images by shifting random grids. These schemes hope all of the pixels on secret images achieve the best utilization and they reduce the quantity of distortion when decrypting the secret images. So, users can get more information by using shifting random grids techniques.

Keywords: Visual Cryptography; Visual Secret Sharing; Random Grids; Multi-Secret Images; Share; Distortion; Codebook.

Rainbow connection numbers of Cartesian product of graphs

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Given a connected graph G together with a coloring f from the edge set of G to a set of colors, where adjacent edges may be colored the same, a u-v path P in G is said to be a *rainbow path* if no two edges of P are colored the same. A u-v path P in G is said to be a *rainbow u-v geodesic* in G if P is a rainbow u-v path whose length equals to the distance of u and v. The graph G is *rainbow-connected*(resp., *strongly rainbow-connected*) if G contains a rainbow u-v path(resp,. rainbow u-v geodesic) for every two vertices u and v of G. In this case, the coloring f is called a *rainbow coloring*(resp,. *strong rainbow coloring*) of G. A rainbow coloring(resp., strong rainbow coloring) of G using k colors is a *rainbow k-coloring*(resp., *strong rainbow k-coloring*) of G. The minimum k for which there exists a rainbow k-coloring(resp., strong rainbow k-coloring) of G is called the *rainbow connection number*(resp., *strong rainbow connection number*) of G and is denoted by $rc(G)$(resp., $src(G)$). We study the rainbow connection numbers and the strong rainbow connection numbers of Cartesian product of graphs, where both of the two graphs are in $\mathcal{F} = \{G : G$ is a path, a cycle, or a complete graph$\}$, or both of the two graphs are in $\mathcal{T} = \{T : T$ is a tree$\}$, in this thesis. We show that if G is the Cartesian product of two graphs G_1 and G_2, in $\mathcal{F}$, then $\mathrm{diam}(G) = rc(G) = src(G)$, except that both G_1 and G_2 are odd cycles. And we prove that if G is the Cartesian product of two trees T_1 and T_2, then $rc(G) = \mathrm{diam}(G)$, except that T_2 is the path P_2, and T_1 satisfies some special conditions, in which case the rainbow connection number of G equals $\mathrm{diam}(G) + 1$.

Keywords: rainbow coloring, strong rainbow coloring, Cartesian product, path, cycle, tree.

Outer-connected domination numbers of block graphs

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Given a graph G, a set S is an *outer-connected dominating set* if every vertex not in S is adjacent to some vertex in S and the subgraph induced by $V \setminus S$ is connected. The *outer-connected domination number* $\tilde{r}_c(G)$ is the minimum size of such a set. In this thesis, we present a linear-time algorithm for the outer-connected domination problem in trees and block graphs, and gives formulas to compute the outer-connected domination numbers of full k-ary trees.

Keywords: dominating set, domination number, outer-connected dominating set, outer-connected domination number, tree, block graph, full k-ary tree..

On Zero-Sum Flows and Flow Numbers of Undirected Graphs

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As an analogous concept of a nowhere-zero flow for directed graphs and a special case of a nowhere-zero flow for bidirected graphs, we consider zero-sum flows for undirected graphs in this thesis. For an undirected graph G, a **zero-sum flow** is an assignment of non-zero integers to the edges such that the sum of the values of all edges incident with each vertex is zero, and we call it a **zero-sum k-flow** if the values of edges are less than k. Also as an analogous concept of minimum flow numbers for directed graphs, we define the **zero-sum minimum flow number** of G as the least positive integer k for which G admits a zero-sum k-flow, and denoted by $F(G)$. On one hand we extend the study of zero-sum integer flows to abelian group flows, and emphasize over the $\mathbb{Z}_k$ case. On the other hand, we extend the concept zero-sum flow to a more general one, namely a **constant-sum flow**. The constant under a constant-sum flow is called an **index** of G, and $I(G)$ is denoted as the **index set** of all possible indices of G. In this thesis, among others we have the following results and most of them have been already published:

Theorem 1 *The zero-sum minimum flow number of F_n graphs are as follows:*

$$F(F_n) = \begin{cases} \infty, & n = 1, 2, 3. \\ 3, & n = 3k+1,\ k \geq 1. \\ 4, & \textit{otherwise}. \end{cases}$$

Theorem 2 *The zero-sum minimum flow number of W_n graphs with $n \geq 3$ are as follows:*

$$F(W_n) = \begin{cases} 5, & n = 5. \\ 3, & n = 3k,\ k \geq 1. \\ 4, & \textit{otherwise}. \end{cases}$$

Theorem 3 *Suppose G be a r-regular graph and $|V(G)| = n$, then for all $r \geq 3$ we have:*

1. $F(G) = 2$ *if* $r \equiv 0 (mod\ 4)$ *or* $r \equiv 2 (mod\ 4)$ *with even* n.
2. $F(G) = 3$ *if* $r \equiv 2 (mod\ 4)$ *with odd* n *or* G *has a 1-factor with odd* r.
3. $F(G) \leq 4$ *if* G *is 2-edge-connected and* $r = 5, 11, 13, 15, 17, \cdots$.
4. $F(G) \leq 5$ *if* $r \neq 5$, *and* $F(G) \leq 7$ *for* G *is 5-regular.*

Theorem 4 *The constant-sum flow index sets of F_n graphs are as follows:*

$$I(G) = \begin{cases} \emptyset, & n = 3. \\ \mathbf{2}\mathbb{Z}, & n = 2k,\ k \geq 2. \\ \mathbb{Z}, & n = 2k+1,\ k \geq 2. \end{cases}$$

Theorem 5 *The constant-sum flow index sets of W_n graphs are as follows:*

$$I(G) = \begin{cases} \mathbf{2}\mathbb{Z}, & n = 2k,\ k \geq 2. \\ \mathbb{Z}, & n = 2k+1,\ k \geq 1. \end{cases}$$

Theorem 6 *The constant-sum flow index sets of r-regular graphs G of order n, are as follows:*

$$I(G) = \begin{cases} \mathbb{Z}^*, & r = 1. \\ \mathbb{Z}, & r = 2 \text{ and } G \text{ contains even cycles only.} \\ \mathbf{2}\mathbb{Z}^*, & r = 2 \text{ and } G \text{ contains an odd cycle.} \\ \mathbf{2}\mathbb{Z}, & r \geq 3,\ r \text{ even and } n \text{ odd}. \\ \mathbb{Z}, & r \geq 3,\ \text{ and } n \text{ even}. \end{cases}$$

Theorem 7 *If G is a $4r$-regular graph with even vertices and connected, then $I_4(G) = \mathbb{Z}_\mathbf{4}$ for all $r \geq 2$, where $I_4(G)$ is the constant-sum $\mathbb{Z}_4$-flow index set of G.*

Keywords: Zero-Sum Flow, Zero-Sum Flow Number, Constant-Sum Flow, Constant-Sum Group Flow, Regular Graph, Eulerian Graph, Fan, Wheel.

The Forbidden Subposet Problems and Turán Problems

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A family $\mathcal{F}$ of subsets of $[n] := \{1, \ldots, n\}$ is said to be P-free if for a given *partially ordered set* (poset) $P = (P, \leq)$, there is no order-preserving injection from $(P, \leq)$ to $(\mathcal{F}, \subseteq)$. The *Lubell function* of a family $\mathcal{F}$ of subsets of $[n]$ is $\bar{h}_n(\mathcal{F}) := \sum_{E \in \mathcal{F}} \binom{n}{|E|}^{-1}$ which can be viewed as the weighted sum of sets in $\mathcal{F}$. We will study the limit of the sequence $\{\lambda_n(P)\}$, where $\lambda_n(P) := \max_{\mathcal{F}} \bar{h}_n(\mathcal{F})$ over all P-free families $\mathcal{F}$ for some posets P and find the connection between this limit and the Turán density of some hypergrpahs.

Keywords: Lubell function, forbidden subposets, Turán density.

Facebook – A smaller world

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"Six degree of Separation" told us: any two individuals, selected randomly from almost anywhere on the planet, can know each other via a chain of average no more than six intermediate acquaintances. There are more tens of millions of people around the world, but the social network is a small world. With the dramatic growth of the World Wide Web and the Internet, even the rise of the social network-Facebook, the distance between two people seems much shorter than before. Through the experiment result, on Facebook, any two individuals are connected in five steps or fewer, on average. The world seems smaller. In this thesis, we construct a dynamic random graph model to simulate Facebook. We regard each user of Facebook as a vertex and the friendship between two users as an edge, and try to depict the pattern of the random graph as time being approximately infinity. In the process of the construction, we applied different probability distributions to adding new vertices and edges, and deleting existing vertices and edges. Based on the preferential attachment and the idea of the weaker tends to be weeded out, the model seems to conform with Facebook. Furthermore, we prove that the degree distribution satisfies the power-law, a common feature of the small world networks. Therefore, we conclude that Facebook is also a small world.

Keywords: Facebook, small world, power law.

A Mathematical Model for Finding the Culprit Who Spreads Rumors

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In this thesis, we introduce a rumor spreading model based on the common susceptible-infected (SI) model which is a well known epidemiological model. We describe the maximum likelihood estimators of graphs and we evaluate the detection probabilities of finding the rumor source in d-regular trees. We observe that: For paths, the detection probability of finding the rumor source scales as $t^{-1/2}$, which approaches 0 as t approaches infinity. For regular trees, we find an explicit bound of the detection probabilities of finding the source in d-regular trees. As a consequence, for $d = 3$, the detection probability approaches 1/4, this result has been obtained earlier by using a random graph model.

Keywords: rumor spreading model, rumor center, detection probability.

The game Grundy arboricity of graphs

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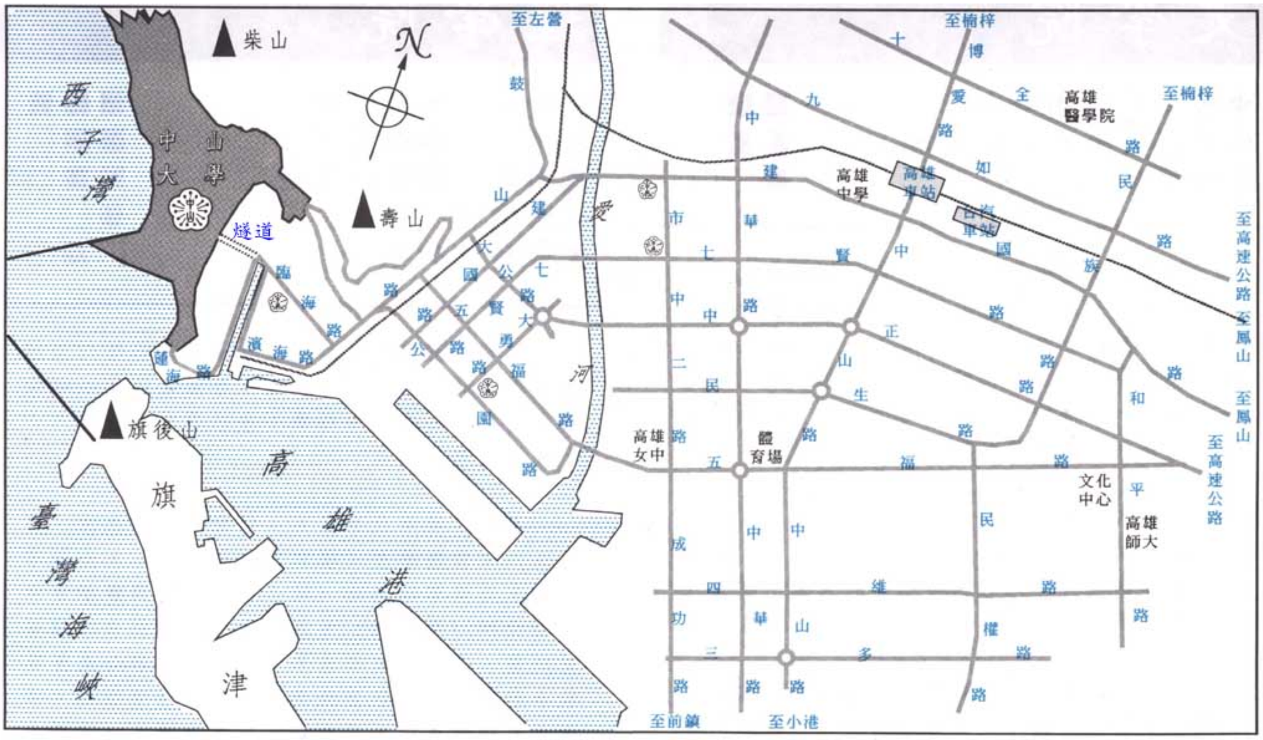
Given a graph $G = (V, E)$, two players, Alice and Bob, alternate their turns to choose an uncolored edges to be colored. Whenever an uncolored edge is chosen, it is colored by the least positive integer so that no monochromatic cycle is created. Alice's goal is to minimize the total number of colors used in the game, while Bob's goal is to maximize it. The game Grundy arboricity of G is the number of colors used in the game when both players use optimal strategies. This thesis discusses the game Grundy arboricity of graphs. It is proved that if a graph G has arboricity k, then the game Grundy arboricity of G is at most $3k - 1$. If a graph G has an acyclic orientation D with maximum out-degree at most k, then the game Grundy arboricity of G is at most $3k - 2$.

Keywords: arboricity, game Grundy arboricity, acyclic orientation, outerplanar graph, coloring game.

學校附近地圖：



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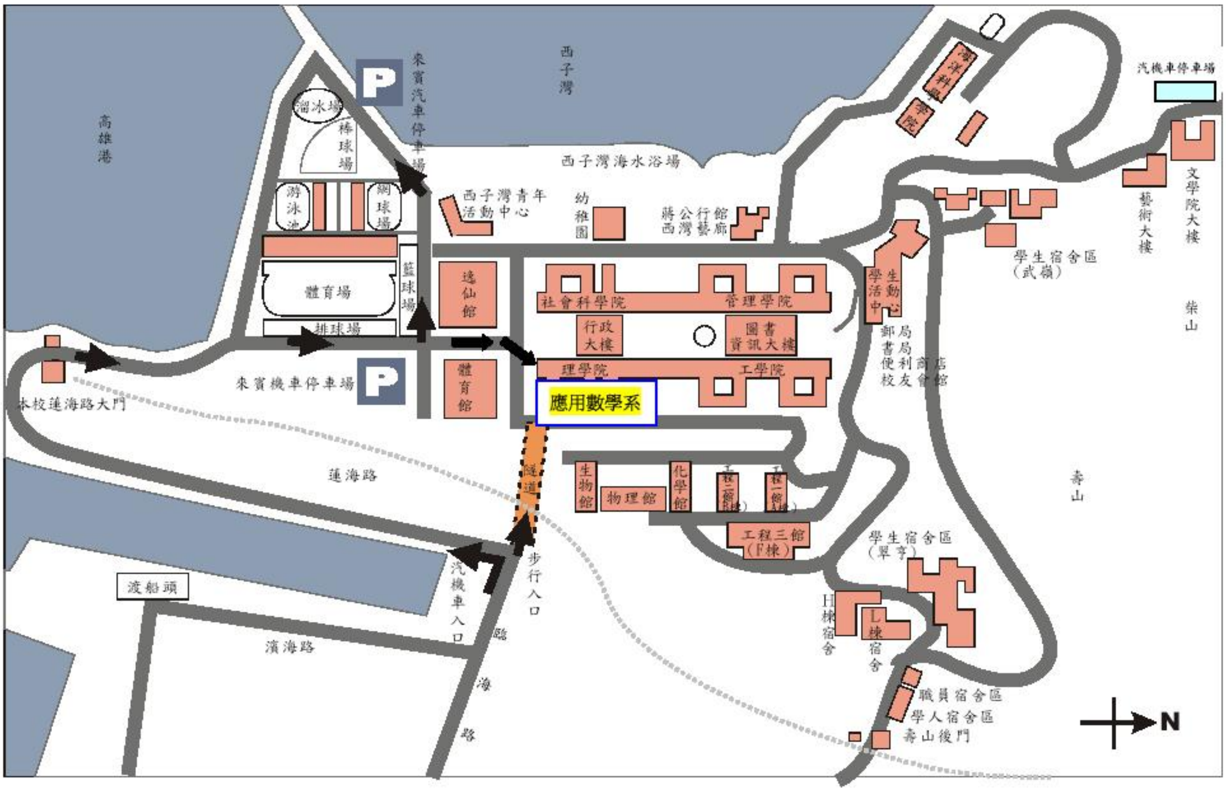

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