# 2013 組合數學新苗研討會

2013 年 08 月 10 日至 2013 年 08 月 11 日

主辦單位:國立高雄師範大學數學系

贊助單位:國科會數學研究推動中心

# 2013 組合數學新苗研討會

## 高雄師範大學數學系

**2013 年 8 月10 (六)至2013 年 8 月11日(日)**

### 邀請講者

傅恆霖 教授 交通大學應用數學系
翁志文 教授 交通大學應用數學系

### 地點

高雄市苓雅區和平一路 116 號
高雄師範大學和平校區活動中心三樓演講廳

### 會議網址

http://cauchy.math.nknu.edu.tw/math/combinatorics/combinatorics.html

贊助單位 國科會數學研究推動中心
主持人 高雄師範大學數學系 賴欣豪 07-7172930 ext 6807
hsinhaolai@nknucc.nknu.edu.tw
會議聯絡 82444 高雄市燕巢區深中路62號 致理大樓6樓607室
高雄師範大學數學系 鄭佩君 07-7172930 ext 6802
s9127@nknucc.nknu.edu.tw

# 2013 組合數學新苗研討會

# 時間表

◎時　間：102 年 08 月 10 日(星期六)~ 102 年 08 月 11 日(星期日)

◎地　點：國立高雄師範大學 和平校區 活動中心 3 樓 演講廳

| | 8 月 10 日 (六) | | 8 月 11 日 (日) |
|---|---|---|---|
| 9:00-10:00 | 報到 | 9:00- 9:30 | 報到 |
| 10:00-10:15 | 開幕 | Session 6 | 主 持 人: 黃國卿 |
| Session 1 | 主 持 人: 李國偉 | 9:30-10:20 | 邀請演講: 翁志文 |
| 10:15-11:05 | 邀請演講: 傅恆霖 | 10:20-10:40 | 休息 |
| 11:05-11:20 | 休息 | Session 7 | 主 持 人: 郭君逸 |
| Session 2 | 主 持 人: 陳宏賓 | 10:40-11:05 | 黃喻培 |
| 11:20-11:45 | 徐育鋒 | 11:05-11:30 | 王稟鈞 |
| 11:45-12:10 | 蕭禕廷 | 11:30-11:55 | 陳立志 |
| 12:10-13:45 | 午餐 | 11:55-13:45 | 午餐 |
| Session 3 | 主 持 人: 蔡秉穎 | Session 8 | 主 持 人: 潘志實 |
| 13:45-14:10 | 吳凱修 | 13:45-14:10 | 虞沛鐸 |
| 14:10-14:35 | 林仁俊 | 14:10-14:35 | 林立庭 |
| 14:35-15:00 | 王文 | 14:35-15:00 | 吳柏翰 |
| 15:00-15:20 | 休息 | 15:00-15:20 | 休息 |
| Session 4 | 主 持 人: 嚴志弘 | Session 9 | 主 持 人: 林武雄 |
| 15:20-15:45 | 蔡孟璇 | 15:20-15:45 | 張光輝 |
| 16:45-16:10 | 王博賢 | 15:45-16:10 | 林紹鈞 |
| 16:10-16:35 | 趙彥丞 | 16:10- | 頒發優秀論文獎 |
| 16:35-16:55 | 休息 | | 賦歸 |
| Session 5 | 主 持 人: 李渭天 | | |
| 16:55-17:20 | 梁育菖 | | |
| 17:20-17:45 | 林哲宇 | | |
| 18:00- | 晚餐 | | |


- **主　辦　單　位：** 國立高雄師範大學 數學系
- **贊　助　單　位：** 國科會數學研究推動中心

# 演講者名單及摘要頁次

## 邀請演講

| 姓名 | 所屬單位 | 題目 | 演講時間 | 頁次 |
|---|---|---|---|---|
| 傅恆霖 | 交通大學應用數學系 | Graph covering and its applications | 星期六 10:15-11:05 | 1 |
| 翁志文 | 交通大學應用數學系 | Spectral graph theory and its applications | 星期日 9:30-10:20 | 2 |

## 學生演講

| 姓名 | 所屬單位 | 題目 | 指導教授 | 演講時間 | 頁次 |
|---|---|---|---|---|---|
| 徐育鋒 | 淡江大學數學系 | Packing graphs with 4-cycles | 高金美 | 星期六 11:20-11:45 | 3 |
| 蕭禕廷 | 交通大學應用數學系 | Mid-labeled partial digest problem | 傅恆霖 | 星期六 11:45-12:10 | 5 |
| 吳凱修 | 暨南國際大學資訊工程學系 | Mutually independent Hamiltonian cycles on Cartesian product graphs | 阮夙姿 | 星期六 13:45-14:10 | 6 |
| 林仁俊 | 東華大學應用數學系 | All-to-all broadcast problems on Cartesian product graphs | 郭大衛 | 星期六 14:10-14:35 | 7 |
| 王文 | 中山大學應用數學系 | Domination number of Cartesian product of graphs | 朱緒鼎 | 星期六 14:35-15:00 | 8 |
| 蔡孟璇 | 東華大學應用數學系 | L(p,q)-labelings of subdivision of graphs | 郭大衛 | 星期六 15:20-15:45 | 9 |

| | | | | | |
|---|---|---|---|---|---|
| 王博賢 | 台灣師範大學數學系 | The chromatic number of the square of a planar graph | 郭君逸 | 星期六 16:45-16:10 | 10 |
| 趙彥丞 | 交通大學應用數學系 | A distance-two coloring with application to wireless sensor and actor networks | 陳秋媛 | 星期六 16:10-16:35 | 11 |
| 梁育菖 | 中山大學應用數學系 | Anti-magic labeling of regular graphs with odd degree | 朱緒鼎 | 星期六 16:55-17:20 | 12 |
| 林哲宇 | 中山大學應用數學系 | Circular chromatic indices of regular graphs | 朱緒鼎 | 星期六 17:20-17:45 | 13 |
| 黃喻培 | 交通大學應用數學系 | Spectral radius and average 2-degree sequence of a graph | 翁志文 | 星期日 10:40-11:05 | 14 |
| 王稟鈞 | 交通大學應用數學系 | The minimum rank of a mountain | 翁志文 | 星期日 11:05-11:30 | 15 |
| 陳立志 | 高雄大學應用數學系 | On skew Fuss paths | 游森棚 | 星期日 11:30-11:55 | 16 |
| 虞沛鐸 | 交通大學應用數學系 | Search for rumor center | 傅恆霖 | 星期日 13:45-14:10 | 17 |
| 林立庭 | 交通大學應用數學系 | The subtree size profile of generalized PORTs and d-ary trees | 符麥克 | 星期日 14:10-14:35 | 18 |
| 吳柏翰 | 清華大學數學系 | A study on the optimal pebbling of graphs | 蔡孟傑/傅恆霖 | 星期日 14:35-15:00 | 19 |
| 張光輝 | 東海大學應用數學系 | On graph labeling problems of magic and antimagic types | 王道明 | 星期日 15:20-15:45 | 20 |
| 林紹鈞 | 交通大學應用數學系 | New adaptive localization algorithms that achieve better coverage for wireless sensor networks | 陳秋媛 | 星期日 15:45-16:10 | 21 |

# Graph Covering and Its Applications

Hung-Lin Fu 傅恆霖
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
hlfu@math.nctu.edu.tw

Let $G$ be a graph. A graph covering $C = \{G_1, G_2, \ldots, G_t\}$ is a collection of $t$ subgraphs of $G$ such that each edge of $G$ occurs in at least one subgraph in $C$. A standard application of graph covering is to find as few isomorphic subgraphs of $G$ as possible. If all subgraphs are isomorphic to $H$, then it is known as minimum $H$-covering. In this talk, I will introduce a couple of applications in which the constraint is on the sum of the orders of the subgraphs and the subgraphs used are not necessarily isomorphic.

# Spectral Graph Theory and Its Applications

Chih-wen Weng 翁志文
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
weng@math.nctu.edu.tw

There are some matrices associated with a graph. The eigenvalues and corresponding eigenvectors of these matrices give hints of the graph properties, and thus have many real-world applications. The talk will start by introducing these applications, and end by introducing the methods to determine or to estimate these eigenvalues and eigenvectors in practice and in theory.

# Packing Graphs with 4-Cycles

Yu-Fong Hsu 徐育鋒
Department of Mathematics
Tamkang University
New Taipei City, Taiwan
joliwugy@yahoo.com.tw

Advisor: Chin-Mei Kau 高金美

A *$k$-cycle* of a graph $G$ is a set $\mathcal{C}$ of edge disjoint $k$-cycles in $G$. A $k$-cycle packing $\mathcal{C}$ of $G$ is *maximum* if $|\mathcal{C}| \geq |\mathcal{C}'|$ for all other $k$-cycle packings $\mathcal{C}'$ of $G$. The *leave* of a $k$-cycle packing of $G$ is the set of edges of $G$ that occur in no $k$-cycle in $\mathcal{C}$; sometimes we alos refer to the subgraph induced by these edges as the leave. A *$k$-cycle system* of $G$ is a $k$-cycle packing of $G$ for which the leave is empty. If $\mathcal{C}$ is a $k$-cycle system of $G$, then $\mathcal{C}$ is the set of edge-disjoint $k$-cycle in $G$ and each edge of $G$ occurs in exactly one of $k$-cycles in $\mathcal{C}$, i.e., $G$ can be decomposed into $k$-cycles.

In Chapter 2, we give some definitions in graph theory and introduce the main tool of the proof we will use in the following chapters.

In Chapter 3, we show that almost all 4-regular graphs of order at least 8 are 3-reducible.

In Chapter 4, we find that some necessary and sufficient conditions for the existence of 4-cycle system of $K_n - E(G)$ as follows, where $G$ is a 4-regular subgraph of order $t$.

(1) If $G$ is a vertex-disjoint union $t/5$ copies of $K_5$ and $G$ is $(n,t)$-admissible, then there exists a 4-cycle system of $K_n - E(G)$ for $n - t \equiv 1 - t/5 \pmod 8$.

(2) If $G$ is $(n,t)$-admissible and $n \geq (4t-7)/3$, then there exists a 4-cycle system of $K_n - E(G)$ for $n \equiv 1, 5 \pmod 8$, except two cases for $n = 9$, $t = 8$.

(3) If $G$ is isomorphic to one of the graphs in Figure 1.1 and $G$ is $(n,t)$-admissible, then there is a 4-cycle system of $K_n - E(G)$ for $n - 17 \equiv 0 \pmod 8$.

In Chapter 5, we summairze the results obtained form the previous chapter and give some problems we have met.

Keywords: Packing, Graph Decomposition, 4-Cycle System, 4-Regular Graph.

# Mid-labeled Partial Digest Problem

Yi-Ting Hsiao 蕭禕廷
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
etonhsiao@yahoo.com.tw

Advisor: Hung-Lin Fu 傅恆霖

In DNA sequencing, the *partial digest problem (PDP)* plays an important role on reconstructing the locations of restriction sites in DNA. From combinatorial point of view, we can model PDP as follow. Let $X = \{x_1, x_2, ..., x_n\}$ be the set of restriction sites with $x_1 = 0$, and $x_1 < x_2 < ... < x_n$ be positive integers, and $\Delta X = \{x_j - x_i \mid 1 \leq i < j \leq n\}$ be the multi-set of distances between every two distinct restriction sites. Then, the PDP is to find the solutions $X$ by knowing $\Delta X$.

Though the model looks simple, so far, the hardness of PDP is still unknown. It seems to us, the PDP is a NP-hard problem. In this thesis, motivated by the approach of *end-labeled partial digest*, *labeled partial digest* and the idea of *probed partial digest*, we propose an idea called *mid-labeled partial digest*. As a consequence, if $k$ mid-labels are added, then we obtain an algorithm with running time $O(n^{\frac{2}{3}} 2^{\frac{2n}{k+1}} \lg n)$ in the worst case.

Keywords: partial digest problem, algorithm.

# Mutually Independent Hamiltonian Cycles On Cartesian Product Graphs

Kai-Siou Wu 吳凱修
Department of Computer Science and Information Engineering
National Chi Nan University
Nantou puli, Taiwan
s100321523@ncnu.edu.tw

Advisor: Justie Su-Tzu Juan 阮夙姿

In a graph $G$, a *cycle* $C = \langle v_0, v_1, ..., v_k, v_0 \rangle$ is defined as a sequence of adjacent vertices and for all $0 \leq i < j \leq k, v_i \neq v_j$; a cycle is called *Hamiltonian cycle* if it contains all vertices of $G$. If there exists a Hamiltonian cycle in $G$, then $G$ is a *Hamiltonian graph*. Two Hamiltonian cycles $C_1 = \langle u_0, u_1, u_2, ..., u_{n-1}, u_0 \rangle$ and $C_2 = \langle v_0, v_1, v_2, ..., v_{n-1}, v_0 \rangle$ are *independent* if $u_0 = v_0, u_i \neq v_i$ for all $1 \leq i \leq n-1$. A set of Hamiltonian cycles $C = \{C_1, C_2, ..., C_k\}$ of $G$ are *mutually independent* if any two different Hamiltonian cycles of $C$ are independent. The *mutually independent Hamiltonianicity* of graph $G$, namely $\text{IHC}(G) = k$, is the maximum integer $k$ such that for any vertex $u$ of $G$ there exists $k$-mutually independent Hamiltonian cycles starting at $u$. The *Cartesian product* of graphs $G$ and $H$, written by $G \times H$, is the graph with vertex set $V(G) \times V(H)$ specified by putting $(u, v)$ adjacent to $(u', v')$ if and only if (1) $u = u'$ and $vv' \in E(H)$, or (2) $v = v'$ and $uu' \in E(G)$.

In this thesis, we study mutually independent Hamiltonianicity on $G = G_1 \times G_2$, where $G_1$ and $G_2$ are Hamiltonian graphs. We prove that $\text{IHC}(G_1 \times G_2) \geq \text{IHC}(G_1)$ or $\text{IHC}(G_1) + 2$ when given some difference conditions. We refer to *toroidal mesh graph* and define Cartesian product of two cycles as $C_m \times C_n$, where $m, n$ are lengths of cycles. We show that $\text{IHC}(C_m \times C_n) = 4$ for any positive integers $m, n \geq 3$.

Keywords: Hamiltonian, Toroidal mesh, Mutually Independent, Hamiltonianicity, Cartesian product.

# All-to-all Broadcast Problems on Cartesian Product Graphs

Jen-Chun Ling 林仁俊
Department of Applied Mathematics
National Dong Hwa University
Hualien, Taiwan
jim200102@hotmail.com

Advisor: David Kuo 郭大衛

All-to-all communication occurs in many important applications in parallel processing. In this thesis, we study the all-to-all broadcast number (the shortest time needed to complete the all-to-all broadcast) of Cartesian product of graphs under the assumption that: each vertex can use all of its links at the same time, and each communication link is half duplex and can carry only one message at a unit of time. We give upper and lower bounds for the all-to-all broadcast number of Cartesian product of graphs and give formulas for the all-to-all broadcast numbers of some classes of graphs, such as the Cartesian product of two cycles, the Cartesian product of a cycle with a complete graph of odd order, the Cartesian product of two complete graphs of odd order, and the hypercube $Q_{2n}$ under this model.

Keywords: all-to-all broadcast, broadcasting set, all-to-all broadcasting number, Cartesian product, cycle, complete graph, hypercube.

# Domination Number of Cartesian Product of Graphs

Wen Wang 王文
Department of Applied Mathematics
National Sun Yat-sen University
Kaohsiung, Taiwan
lainmidi@gmail.com

Advisor: Xuding Zhu 朱緒鼎

For a graph $G$, $\gamma(G)$ is the domination number of $G$. Vizing conjectured that $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ for any graph $G$ and $H$, where $G\Box H$ is the Cartesian product of graphs $G$ and $H$. Clark and Suen proved that $\gamma(G\Box H) \geq \frac{1}{2}\gamma(G)\gamma(H)$ for any graphs $G$ and $H$.

We prove that if $G$ has $S_1, S_2, .., S_k$ disjoint sets of $V(G)$, for any $D \subseteq V(G)$, we have $|D| + |B_D| \geq k$ where $B_D = \{\ i\ |S_i \nsubseteq N[D], S_i \cap D = \emptyset\}$, then $\gamma(G\Box H) \geq k\ \gamma(H)$.

Keywords: Domination Number, Cartesian Product, Vizing conjectured.

# $L(p,q)$-labelings of Subdivision of Graphs

Meng-Hsuan Tsai 蔡孟璇
Department of Applied Mathematics
National Dong Hwa University
Hualien, Taiwan
shiuan990205@gmail.com

Advisor: David Kuo 郭大衛

Given a graph $G$ and a function $h$ from $E(G)$ to $\mathbb{N}$, the $h$-subdivision of $G$, denoted by $G_{(h)}$, is the graph obtained from $G$ by replacing each edge $uv$ in $G$ with a path $P : ux_{uv}^1 x_{uv}^2 \dots x_{uv}^{n-1} v$, where $n = h(uv)$. Given a graph $G$ and two positive integers $p, q$ with $p > q$ an $L(p,q)$ -labeling of $G$ is a function $f$ from the vertex set $V(G)$ to the set of all nonnegative integers such that $|f(x) - f(y)| \geq p$ if $d_G(x,y) = 1$ and $|f(x) - f(y)| \geq q$ if $d_G(x,y) = 2$. A $k$-$L(p,q)$-labeling is an $L(p,q)$-labeling such that no label is greater than $k$. The $L(p,q)$-labeling number of $G$, denoted by $\lambda_{p,q}(G)$, is the smallest number $k$ such that $G$ has a $k$-$L(p,q)$-labeling.

Keywords: $L(2,1)$-labeling, $L(p,q)$-labeling,subdivision.

# The Chromatic Number of the Square of a Planar Graph

Po-Hsien Wang 王博賢
Department of Mathematics
National Taiwan Normal University
Taipei, Taiwan
pili7494089@yahoo.com.tw

Advisor: Junyi Guo 郭君逸

Discharging method was proposed in 1977 by Appel, he used it to prove that for any planar graph $G$, $\chi(G) \leq 4$, that is well-known 4-Color Theorem. The Heuvel et al. used discharging method to prove $\chi(G^2) \leq 2\Delta + 25$ in 1999. In this paper, we reduce this upper bound to $2\Delta + 10$. And we also generalize the result to $\lambda(G; p, q) \leq (4q - 2)\Delta + 10p + 8q - 9$.

Keywords: planar graph, chromatic number, labeling of a graph, discharging.

# A Distance-two Coloring with Application to Wireless Sensor and Actor Networks

Yen-Cheng Chao 趙彥丞
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
chaopeter34@gmail.com

Advisor: Chiuyuan Chen 陳秋媛

Wireless sensor networks (WSNs) have a wide array of applications in environment and infrastructure monitoring. An efficient solution to allow sensors to communicate with the outside world is making use of one or several actors as the receiver of the data harvested by the WSNs. A wireless sensor and actor network (WSAN) consists of many randomly deployed sensors and a few actors that organize the sensors in their vicinity into an actor-centric network. Localization, routing, and collision avoidance are three fundamental problems in WSANs. The main contribution of this thesis is to solve the collision avoidance problem by proposing a new virtual infrastructure for the localization, and give optimal (in some cases, near-optimal) distance-two colorings for the adjacency graph of our virtual infrastructure.

Keywords: Wireless sensor and actor network, Coarse-grain localization, Distance-two coloring, Collision avoidance.

# Anti-magic Labeling of Regular Graphs with Odd Degree

Yu-Chang Liang 梁育菖
Department of Applied mathematics
National Sun Yat-Sen University
Kaohsiung, Taiwan
chase2369219@hotmail.com

Advisor: Xuding Zhu 朱緒鼎

An anti-magic labeling of a graph $G$ is a one-to-one correspondence between $E(G)$ and $\{1, 2, \ldots, |E|\}$ such that the sum of the labels assigned to edges incident to distinct vertices are different. If $G$ has an anti-magic labeling, then we say $G$ is anti-magic. This paper proves that for any regular graphs with odd degree are anti-magic.

Keywords: Anti-magic, regular graph.

# Circular Chromatic Indices of Regular Graphs

Cheyu Lin 林哲宇
Department of Applied Mathematics
National Sun Yat-sen University
Kaohsiung, Taiwan
showming.lin@gmail.com

Advisor: Xuding Zhu 朱緒鼎

The *circular chromatic index* of a graph $G$, written $\chi'_c(G)$, is the minimum $r$ permitting a function $c$: $E(G) \to [0, r)$ such that $1 \le |c(e) - c(e')| \le r - 1$ whenever $e$ and $e'$ are adjacent. It is known that for any $\epsilon \in (0, 1/3)$, there is a 3-regular simple graph $G$ with $\chi'_c(G) = 3 + \epsilon$. This paper proves the following results: Assume $n \ge 5$ is an odd integer. For any $\epsilon \in (0, 1/4)$, there is an $n$-regular simple graph $G$ with $\chi'_c(G) = n + \epsilon$. For any $\epsilon \in (0, 1/3)$, there is an $n$-regular multigraph $G$ with $\chi'_c(G) = n + \epsilon$; Assume $n \ge 4$ is an even integer. For any $\epsilon \in (0, 1/6)$, there is an $n$-regular simple graph $G$ with $\chi'_c(G) = n + \epsilon$. For any $\epsilon \in (0, 1/3)$, there is an $n$-regular multigraph $G$ with $\chi'_c(G) = n + \epsilon$.
Joint work with Tsai-Lien Wong and Xuding Zhu.

Keywords: circular chromatic index, regular graph.

# Spectral Radius and Average 2-Degree Sequence of a Graph

Yu-pei Huang 黃喻培
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
pei.am91g@nctu.edu.tw

Advisor: Chih-wen Weng 翁志文

In a simple connected graph, the average 2-degree of a vertex is the average degree of its neighbors. With the average 2-degree sequence and the maximum degree ratio of adjacent vertices, we present a sharp upper bound for the maximum eigenvalue of the adjacency matrix of a graph.

Keywords: spectral radius, average 2-degree.

# The Minimum Rank of a Mountain

Ping-Chun Wang 王稟鈞
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
shadow.ktc@msa.hinet.net

Advisor: Chih-wen Weng 翁志文

Let $G$ be a simple graph with vertex set $V(G) = [n] = \{1, 2, \ldots, n\}$ and edge set $E(G)$. The **minimum rank** $m(G)$ of $G$ is the minimum possible rank of an $n$ by $n$ symmetric matrix $A$ whose $ij$-th entry is not zero if and only if $ij \in E(G)$, where $i, j$ are distinct. For $m < n$, a graph $G$ with vertex set $[n]$ is called a **mountain based on** $[m]$ if $G$ satisfies

(i) the subgraph of $G$ induced on $\{1, 2, \ldots, m\}$ is a path which is partitioned into a few segments;

(ii) each segment is assigned a unique vertex in $[n]\backslash[m]$ which has at least two neighbors in the segment; and

(iii) all edges of $G$ are either described in (i) or in (ii).

In the thesis we show that a mountain based on $[m]$ has minimum rank $m - 1$.

Keywords: graph, minimum rank, mountain.

# On Skew Fuss Paths

Li-Chih Chen 陳立志
Department of Applied Mathematics
National University of Kaohsiung
Kaohsiung, Taiwan
li.chih817@gmail.com

Advisor: Sen-Peng Eu 游森棚

It is well known that the number of Dyck paths, which are the paths in the first quadrant using up-steps $(1,1)$, down-steps $(1,-1)$, starts from $(0,0)$ and ends on $x$-axis, is counted by the Catalan numbers. Dyck paths can be generalized to the $m$-Fuss paths, which the allowable steps are up-steps $(m,m)$ and down-step $(1,-1)$. It is also known that the number of $m$-Fuss paths with $n$ up-steps is counted by the $m$-Fuss number $\frac{1}{mn+1}\binom{(m+1)n}{n}$. In 2010, the concept of Dyck paths is generalized to the skew Dyck paths by Deutsch et al. A skew Dyck path is a lattice path in the first quadrant using up-steps $(1,1)$, down-steps $(1,-1)$ and left-step $(-1,-1)$, starting from $(0,0)$ and ending on $x$-axis. Deutsch et al. enumerated the number of skew Dyck paths and found many properties. In this paper we define the *skew Fuss paths*, in which the allowable steps are up-steps $(m,m)$, down-steps $(1,-1)$ and left-step $(-1,-1)$. Our definition of skew Fuss paths is a simultaneous generalization of the Fuss paths and skew Dyck paths. We enumerate the skew Fuss paths and derive some of their properties.

Keywords: Dyck paths, $m$-Fuss paths, skew Dyck paths, skew Fuss paths.

# Search for Rumor Center

Pei-Duo Yu 虞沛鐸
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
joba_ joba2@hotmail.com

Advisor: Hung-Lin Fu 傅恆霖

In the modern world there are many network risks which share a common structure: an isolated risk is amplified because it is spread by the network. In essence, all of these types of spreading phenomenon can be modeled as a *rumor* spreading through a network, where the goal is to find the source of the rumor in order to control and prevent these network risks based on limited information about the network structure and the *rumor infected* nodes.

In this thesis, we shall use the so-called *Rumor Spread* model which is simplified from an epidemic model called *Susceptible-Infected-Recovered model* to study the *Rumor Center* in a tree-shaped network. Several new results are obtained on the cases where the network is defined on a *d-regular* tree either infinite or finite.

Keywords: rumor spreading model, rumor center, detection probability.

# The Subtree Size Profile of Generalized PORTs and $d$-ary Trees

Li-Ting Lin 林立庭
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
andy_00435@hotmail.com

Advisor: Michael Fuchs 符麥克

Generalized plane-oriented recursive trees (PORTs) and $d$-ary trees are two important tree families with many applications. Their properties can be analyzed with the same tools, where properties which are often studied include: the number of nodes of a fixed outdegree, the number of nodes of a fixed level, the number of nodes with subtree rooted at the node having a fixed size, etc.

In this thesis, we will consider the number of nodes with subtree rooted at the node having a fixed size. This is the so called subtree size profile. We will show how to use singularity analysis to obtain the mean value, variance, higher moments and limiting distribution of the subtree size profile for the above two families of trees (under a suitable random model).

Keywords: subtree size profile ,singularity analysis, generalized PORTs, $d$-ary trees.

# A Study on the Optimal Pebbling of Graphs

Wu Bo-Hen 吳柏翰
Department of Mathematics
National Tsing Hua University
Hsinchu, Taiwan
qwaszx06036@gmail.com

Advisor: Meng-Kiat Chuah 蔡孟傑 and Hung-Lin Fu 傅恆霖

A *pebbling move* consists of removing two pebbles from one vertex, throwing one away, and moving the other pebble to an adjacent vertex. A *configuration or distribution* $C$ of a graph $G$ is a mapping from $V(G)$ into the set of nonnegative integers. We let $C(v)$ be the number of pebbles distributed to $v \in V(G)$. So the number of pebbles on $G$ is $\sum_{v \in V(G)} C(v)$. If a configuration $C$ let us move at least one pebble to each vertex $v$ by applying pebbling moves repeatedly(if necessary), then $C$ is called a *pebbling* of $G$. The *optimal pebbling number* $f'(G)$ of $G$ is the minimum number of pebbles used in a pebbling of $G$.

There are many ways to deal with this problem, such as probability [6], error-correcting codes [9] and special types of domination. In this thesis, we set forth the study of pebbling problem on graphs. Mainly, we focus on the optimal pebbling number of the product of two graphs, and the exact values of $f'(P_3 \times P_n)$, $f'(Q_3) = f'(P_2 \times Q_2) = f'(P_2 \times C_4)$, $f'(Q_4) = f'(P_2 \times Q_3)$ and $f'(Q_5) = f'(P_2 \times Q_4)$ are obtained. For more general graphs, upper bounds are provided. We also give a different approach to prove the values of $f'(P_n)$ and $f'(P_2 \times P_n)$.

Keywords: pebbling move, configuration, pebbling, optimal pebbling number.

# On Graph Labeling Problems of Magic and Antimagic Types

Guang-Hui Zhang 張光輝
Department of Applied Mathematics
Tunghai University
Taichung, Taiwan
sincostanzhang@gmail.com

Advisor : Tao-Ming Wang 王道明

In this thesis we consider finite, simple and undirected graphs, unless otherwise stated. For a graph $G$=$(V, E)$ we denote the set of vertices $V(G)$ and the set of edges $E(G)$. We also denote $|V(G)|$=$p$ and $|E(G)|$=$q$. A labeling of graph $G$ is any mapping that sends certain of graph elements to certain set of positive integers or colors. Let $N(u)$ be the set of all neighbors of a given vertex $v$. For an edge labeling $g : E(G) \rightarrow \{1, 2, \cdots, q\}$ of $G$, the associated vertex-weight of a vertex $u \in V(G)$ is $w_g(u) = \sum_{v \in N(u)} \; g(uv)$ and for a total labeling $f : V(G) \cup E(G) \rightarrow \{1, 2, \cdots, p+q\}$ the associated vertex-weight of a vertex $u \in V(G)$ is $w_{gt}(u) = f(u) + \sum_{v \in N(u)} \; g(uv)$. An anti-magic labeling with $q$ edge is a bijection from the set of edge such that all $N(u)$ are distinct. A graph is called anti-magic if it admits an anti-magic labeling. We usually call an anti-magic, **VAE** for short. And **(a,d)-VAE** if $N(u)$ are distinct and all vertex-weights in $G$ is $\{a, a+d, \cdots, a+(p-1)d\}$, where $a > 0$ and $d \geqslant 0$ are two fixed integers. We prove an odd regular graph $G$ is antimagic if $G$ contains a pseudo-prism 3-regular subgraph. Moreover an even regular graph $G$ is antimagic if $G$ contains certain special 2-factors. Furthermore we use the **(a,d)-VAE** result to make super-edge labeling **(E-Sup)** and vertex-magic-total labeling **(VMTL)**, finally we similarly use the constant sum way to prove some zero-sum-flow **(ZSF)** results.

Keywords: antimagic, 2-factor, zero-sum-flow.

# New Adaptive Localization Algorithms That Achieve Better Coverage for Wireless Sensor Networks

Shao-Chun Lin 林紹鈞
Department of Applied Mathematics
National Chiao Tung University
Hsinchu, Taiwan
jk13d.net@gmail.com

Advisor: Chiuyuan Chen 陳秋媛

Many applications in wireless sensor networks (such as forest fires detection, environment monitoring, military, and wildlife tracking) require position information to detect and record events. How to obtain the position information of the sensors become an important problem and is usually called the localization problem. Using Global Positioning System (GPS) is a possible solution for localization but is impractical. The reason is that a wireless sensor network usually has thousands of sensors and it is too expensive to equip every sensor a GPS. In [3], Huang et al. studied a new optimization problem, minimum cost localization problem, which aims to localize all sensors by using the minimum number of anchors. The purpose of this thesis is to propose adaptive algorithms for the localization problem. Our algorithms are simpler than the algorithms in [3] and cover all the cases in the algorithms in [3]; simulation results also show that our algorithms have better coverage.

Keywords: wireless sensor network, localization, algorithm, rigidity, globally rigid, trilateration, triangulation.

# 國立高雄師範大學和平校區平面圖